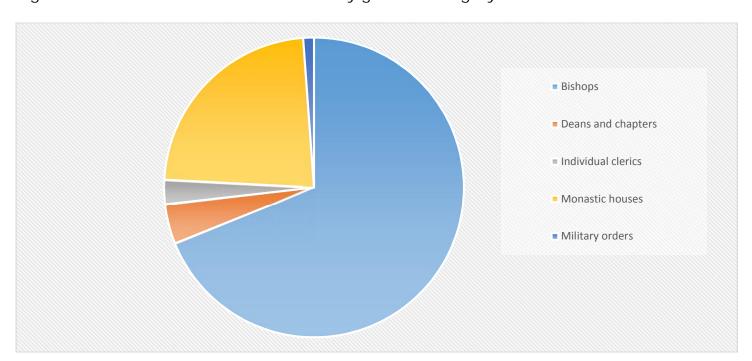
6 CO-WITNESSING NETWORKS: OTHER GRANTOR CATEGORIES

PART ONE: NETWORK OF ALL ECCLESIASTICAL (H2) GRANTORS

The Social Network Analysis of all ecclesiastical documents includes 608 documents, 2009 witnesses, and 19,757 edges. This study is based on the five specified document types in sources with an H-number beginning with 2. Of the 608 documents, 594 (97.7%) of which are charters. There are also 13 notifications and one charter/brieve. Agreements and settlements are not included in the study, as they are H4 documents. Of the 608 documents, over two-thirds (419) are episcopal, with a further 26 documents issued by deans and chapters or individual members of chapters. There are a further 16 documents in the names of various clerks, chaplains, magistri, and other individuals. 23% of the documents were in the names of heads and other members of religious houses, and there were a further 7 documents from military orders. Within the episcopal category, 194 were documents of the bishops of St Andrews, accounting for 46% of episcopal documents, while the bishops of Glasgow accounted for a further 62 documents. Of the religious houses, 41 were from St Andrews priory, 21 from Kelso abbey, and 14 from Arbroath abbey.

Figure 6.1. H2 ecclesiastical documents by grantor category



There were 2009 witnesses in the dataset, of whom only two were women. There were 24 institutions, and the remaining 1983 were individual men.

Figure 6.2. Netdraw: witnesses to H2 documents

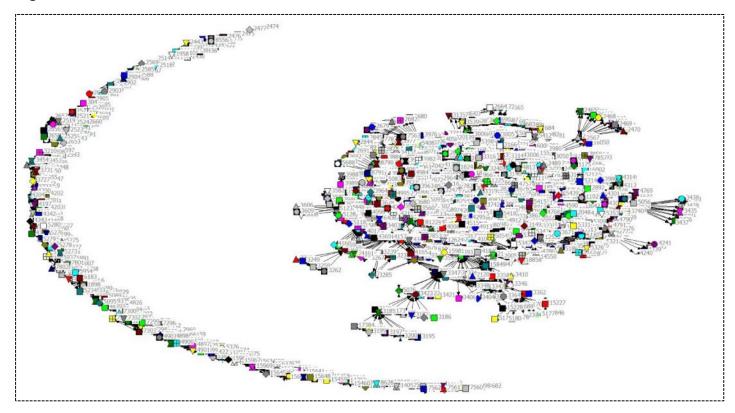
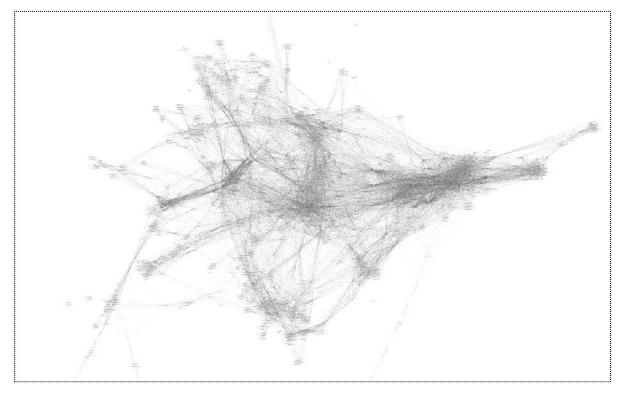


Figure 6.3. Gephi: witnesses to H2 documents



The king's clerk and chancellor William del Bois, who has the highest betweenness centrality in the study of all witnesses (see chapter 4), also has the highest betweenness among all H2 documents (see Table 6.1). As the greatest concentration of individuals is based around the diocese of St Andrews, and the sociogram breaks into various divergent segments by the level of more than 3 co-witnessing acts (see Figure 6.4, below), most of the figures with high betweenness were active in the diocese of St Andrews, including the archdeacons Laurence of Thornton [825], Matthew bishop of Aberdeen [2] (archdeacon of St Andrews from ca 1150 to 1172), Master Adam of Makerstoun [3350], provost of the cell De house of St Mary's on the Rock, Bishop William Malveisin's clerk and chaplain, Peter [2971]. The second most important element in the structure of the sociogram are those people based around the diocese of Glasgow, such as Bishop Walter of St Albans, a previous royal chaplain [858], and the archdeacon Robert [797]. The only lay witnesses with high betweenness were Duncan (II) earl of Fife [13] and Gilbert earl of Strathearn [260]. Finally, it is important to note that the predominance of the secular church over the monastic is a reflection of the makeup of the documentary corpus, as described above.

Table 6.1. Top twenty by betweenness centrality (H2)

Person ID	Name	Betweenness Centrality
42	William del Bois, chancellor (d.1232)	126064.1196
835	Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	95497.14191
2	Matthew, bishop of Aberdeen (d.1199)	92825.68178
2971	Peter, chaplain and clerk of Bishop Malveisin	77211.85245
3350	Adam of Makerstoun, master, provost (d.1280x86)	74011.29714
3871	Edward Murray, master, canon, bishop's clerk	67278.2338
2762	Henry, archdeacon of Dunkeld (fl.1183x1203-1220x25)	63200.2405
13	Duncan (II), earl of Fife (d.1204)	61723.18072
850	John Scott, bishop of Dunkeld (d.1203)	59962.36012
858	Walter of St Albans, bishop of Glasgow (d.1232)	59013.85368
926	Elias of Partick, clerk, canon (son of Fulbert)	58299.16776
797	Robert, archdeacon of Glasgow (d.1222)	51145.2434
829	Ranulf de Wat, archdeacon of St Andrews (d.1209)	50012.20541
2044	Walter de Mortimer, dean of Glasgow (d.1270x71)	44811.84647
3521	Adam de Prebenda, dean of Dunkeld (fl.1229x36-45)	41021.18185
859	Alexander de St Martin, master (fl.1214x40-1247)	40858.55692
3337	Simon Wallace, master, official of St Andrews	39985.35595
776	John of Huntingdon, master, official of Glasgow (fl.1179x1208)	37409.88544
260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	36061.66232
3250	Nicholas of Moffat, archdeacon of Teviotdale (d.1270)	35795.98353

Figure 6.4. Netdraw: witnesses to H2 documents, >3 co-witnessing instances

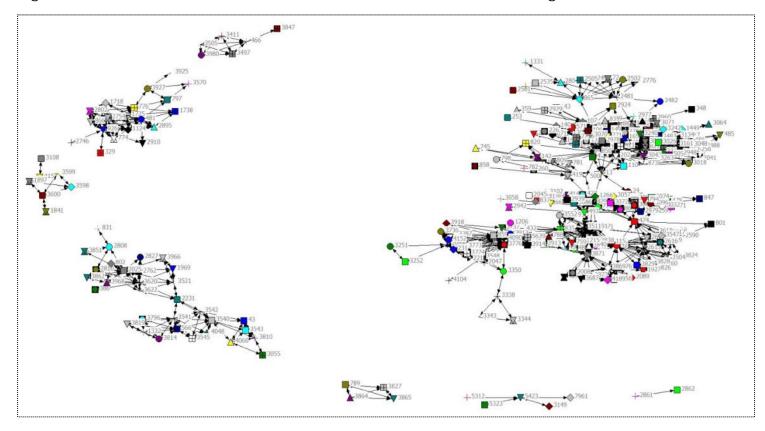


Figure 6.5. Netdraw: witnesses to H2 documents, >5 co-witnessing instances

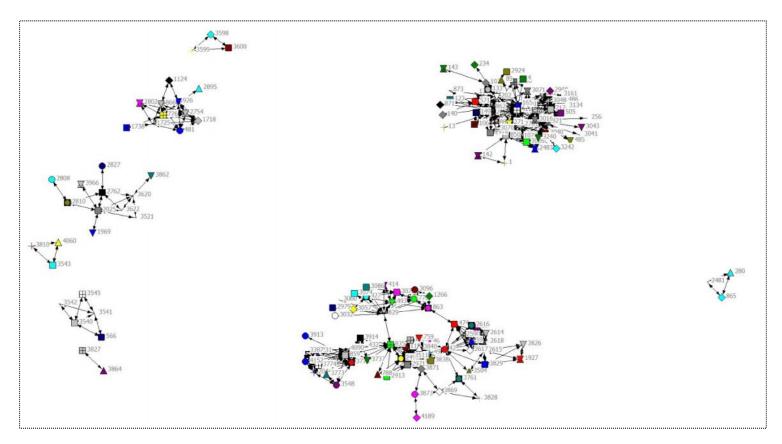


Figure 6.6. Netdraw: witnesses to H2 documents, >7 co-witnessing instances

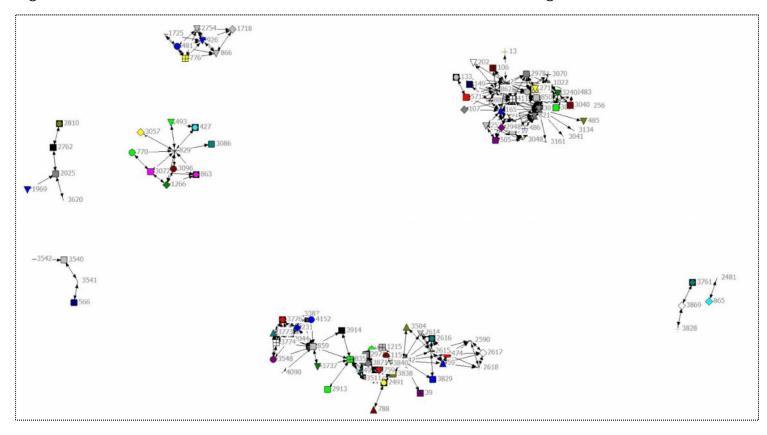


Figure 6.7. Netdraw: witnesses to H2 documents, >10 co-witnessing acts

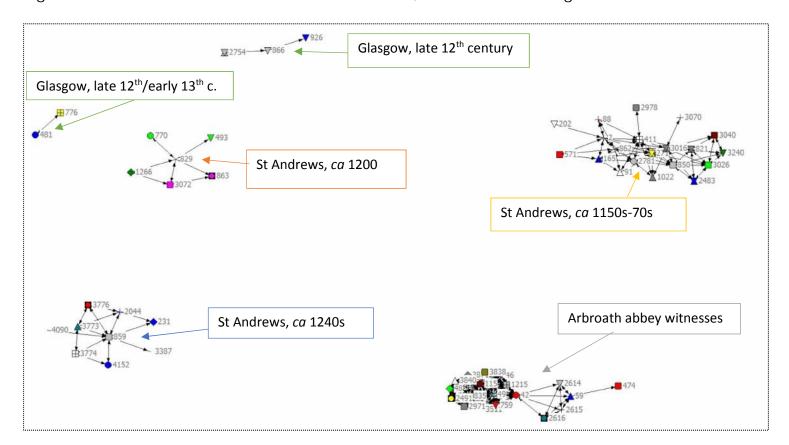


Figure 6.8. Netdraw: witnesses to H2 documents, >12 co-witnessing acts

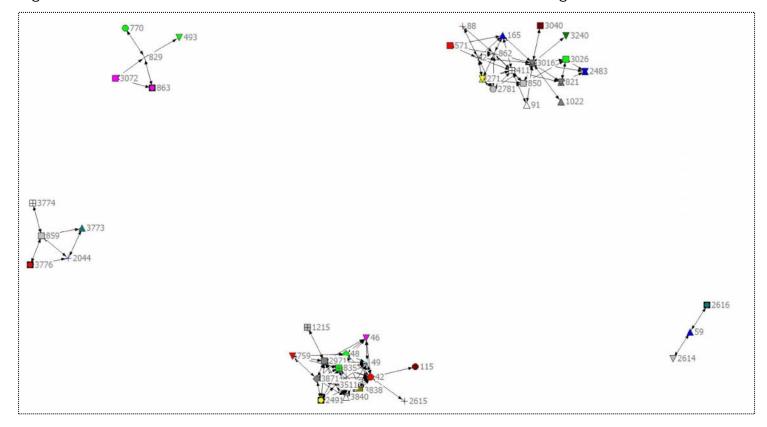


Figure 6.9. Netdraw: witnesses to H2 documents, >15 co-witnessing acts

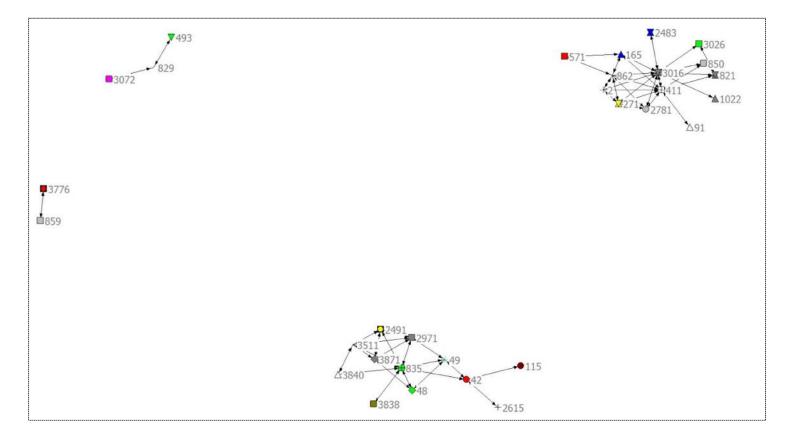


Figure 6.10. Netdraw: witnesses to H2 documents, >17 co-witnessing acts

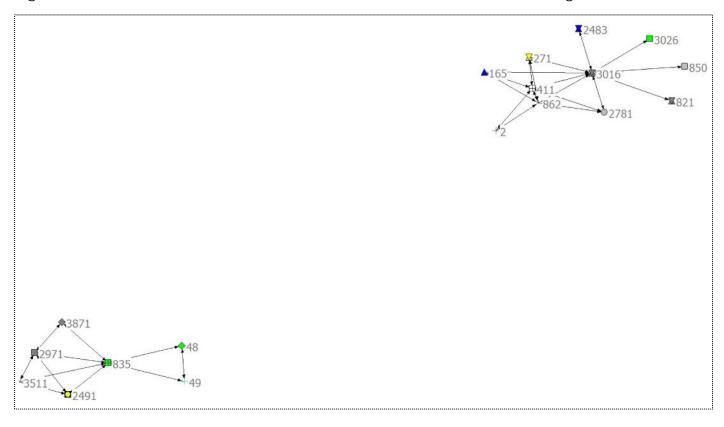


Figure 6.11. Netdraw: witnesses to H2 documents, >20 co-witnessing acts

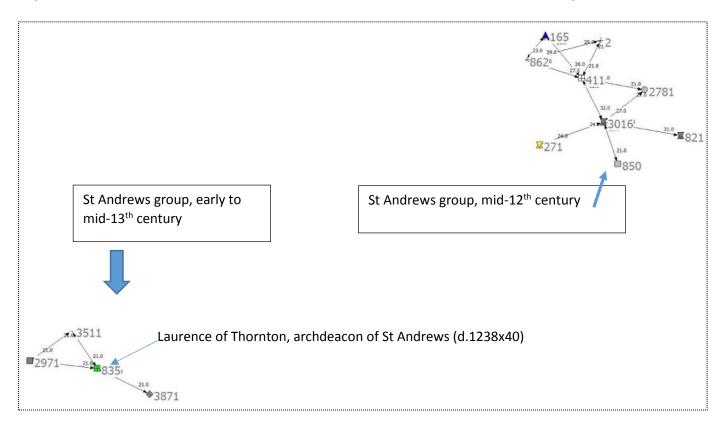


Figure 6.12. Netdraw: witnesses to H2 documents, >21 co-witnessing acts

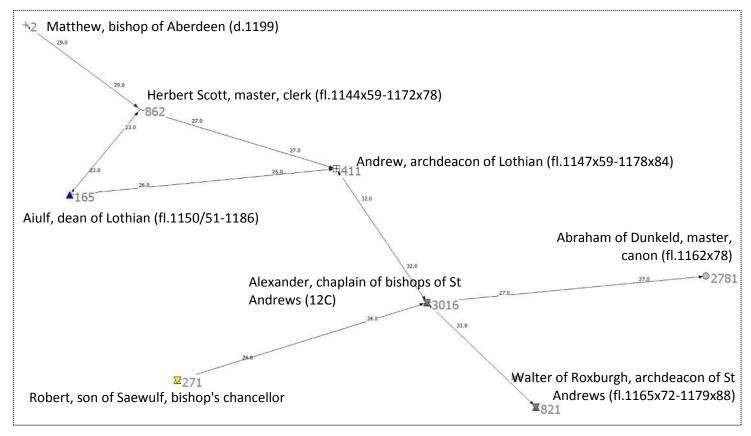


Figure 6.13. Netdraw: witnesses to H2 documents, >25 co-witnessing acts

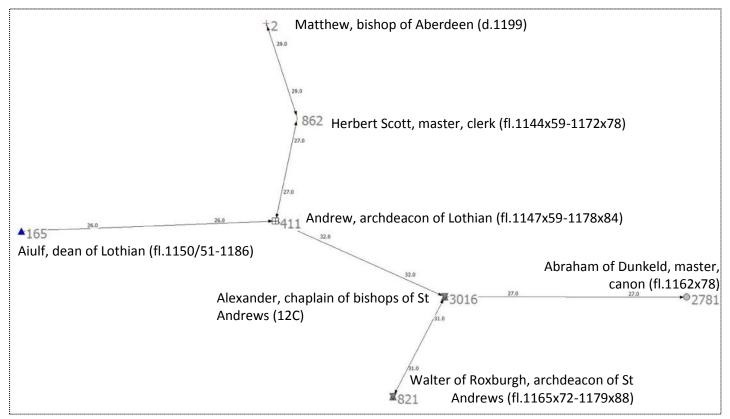


Figure 6.14. Netdraw: witnesses to H2 documents, >30 co-witnessing acts

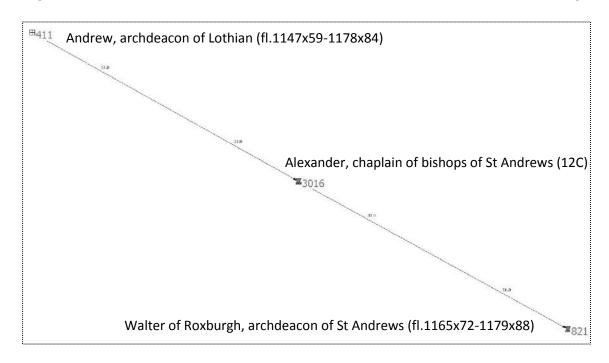


Table 6.2: Most productive co-witnessing 'relationships' (H2)

Person 1	Person 2	# docs
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Alexander, chaplain of bishops of St Andrews (12C)	32
Walter of Roxburgh, archdeacon of St Andrews (fl.1165x72-		
1179x88)	Alexander, chaplain of bishops of St Andrews (12C)	31
Matthew, bishop of Aberdeen (d.1199)	Herbert Scott, master, clerk (fl.1144x59-1172x78)	29
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Herbert Scott, master, clerk (fl.1144x59-1172x78)	27
Abraham of Dunkeld, master, canon (fl.1162x78)	Alexander, chaplain of bishops of St Andrews (12C)	27
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Aiulf, dean of Lothian (fl.1150/51-1186)	26
Robert, son of Saewulf, bishop's chancellor	Alexander, chaplain of bishops of St Andrews (12C)	24
Aiulf, dean of Lothian (fl.1150/51-1186)	Herbert Scott, master, clerk (fl.1144x59-1172x78)	23
Matthew, bishop of Aberdeen (d.1199)	Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	21
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Abraham of Dunkeld, master, canon (fl.1162x78)	21
John Scott, bishop of Dunkeld (d.1203)	Alexander, chaplain of bishops of St Andrews (12C)	21
	Laurence of Thornton, archdeacon of St Andrews	
Peter, chaplain and clerk of Bishop Malveisin	(d.1238x40)	21
Peter, chaplain and clerk of Bishop Malveisin	Michael, master, clerk, chaplain (fl.1201-1220x25)	21
Laurence of Thornton, archdeacon of St Andrews		
(d.1238x40)	Edward Murray, master, canon, bishop's clerk	21
Laurence of Thornton, archdeacon of St Andrews		
(d.1238x40)	Michael, master, clerk, chaplain (fl.1201-1220x25)	21
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Robert, son of Saewulf, bishop's chancellor	20
Peter, chaplain and clerk of Bishop Malveisin	Stephen of Lilliesleaf, master, clerk, persona	20
Herbert Scott, master, clerk (fl.1144x59-1172x78)	Robert, son of Saewulf, bishop's chancellor	20
Laurence of Thornton, archdeacon of St Andrews		
(d.1238x40)	Stephen of Lilliesleaf, master, clerk, persona	20

The table of the most productive co-witnessing 'relationships' reflects the predominance of evidence emanating from St Andrews in the mid-to-late twelfth century. As the above sociograms illustrate, all of the individuals who witnessed alongside each other more than twenty times did so in the context of the church of St Andrews. The chaplain Alexander, various archdeacons of St Andrews and of Lothian, and magistri like Herbert Scot and Abraham of Dunkeld were among the most frequent witnesses.

Table 6.3 (below) lists the top 20 witnesses according to degree centrality. These mostly reflect again the predominance of St Andrews material. Bishop Matthew of Aberdeen, archdeacon of St Andrews from around 1150 to 1172, was connected to 233 other contacts, and was one of the most central individuals in the database as a whole. John Scot was his relative and an unsuccessful nominee for the episcopate in 1178: he later served as bishop of Dunkeld for about two decades. The list again demonstrates the key position of archdeacons, with Ranulf de Wat, Andrew, Thorald, Walter of Roxburgh, and Laurence of Thornton all present on the list. Not all central individuals were St Andrews churchmen themselves: Duncan (II) earl of Fife, the chancellor William del Bois, and Andrew bishop of Caithness, all men with high centralities in the sociogram of all witnesses to the database, witnessed a number of ecclesiastical charters as well as the royal charters we tend to associate them with.

Table 6.3. Top 20 witnesses by degree

poms id	name	degree
2	Matthew, bishop of Aberdeen (d.1199)	233
850	John Scott, bishop of Dunkeld (d.1203)	184
829	Ranulf de Wat, archdeacon of St Andrews (d.1209)	179
3016	Alexander, chaplain of bishops of St Andrews (12C)	178
411	Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	171
165	Aiulf, dean of Lothian (fl.1150/51-1186)	163
13	Duncan (II), earl of Fife (d.1204)	160
862	Herbert Scott, master, clerk (fl.1144x59-1172x78)	145
571	Thorald, archdeacon of Lothian (d.1163 or 1166)	140
493	John of Leicester, bishop of Dunkeld (d.1214)	139
271	Robert, son of Saewulf, bishop's chancellor	137
42	William del Bois, chancellor (d.1232)	133
202	Andrew, bishop of Caithness (d.1184)	129
821	Walter of Roxburgh, archdeacon of St Andrews (fl.1165x72-1179x88)	127
2781	Abraham of Dunkeld, master, canon (fl.1162x78)	126
1022	Odo of Kinninmonth, steward, marischal (d.c.1195)	125
2971	Peter, chaplain and clerk of Bishop Malveisin	120
863	Isaac Scott, master, clerk	119
835	Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	114
776	John of Huntingdon, master, official of Glasgow (fl.1179x1208)	112

Table 6.4 lists the top twenty witnesses according to eigenvector centrality. In addition to having the most contacts (degree), Bishop Matthew of Aberdeen also sets the benchmark in terms of number of high-degree contacts (eigenvector). Bishop John Scot also comes in second here, with an eigenvector score of about 93%, but Earl Duncan II of Fife is number three, with an eigenvector score of 86.7%. So even though his co-witnessing contacts were fewer than Ranulf de Wat or Alexander the chaplain, the individuals with whom he co-witnessed were themselves more central players. This is likely because Earl Duncan would have witnessed at more large assemblies, while St Andrews insiders like Alexander the chaplain would have witnessed more workaday charters in St Andrews. Another point to note is the presence of heads of religious houses and bishops in the 50-60% range. Abbots of Dunfermline and Holyrood and bishops of Dunkeld probably witnessed fewer charters than many others, but these would have likewise been alongside the more prominent power players in the kingdom and the church.

Table 6.4. Top 20 witnesses by eigenvector

Person ID	Name	Eigenvector Centrality
2	Matthew, bishop of Aberdeen (d.1199)	1
850	John Scott, bishop of Dunkeld (d.1203)	0.929662
13	Duncan (II), earl of Fife (d.1204)	0.867203
411	Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	0.795135
3016	Alexander, chaplain of bishops of St Andrews (12C)	0.753157
165	Aiulf, dean of Lothian (fl.1150/51-1186)	0.745829
829	Ranulf de Wat, archdeacon of St Andrews (d.1209)	0.74413
202	Andrew, bishop of Caithness (d.1184)	0.687737
271	Robert, son of Saewulf, bishop's chancellor	0.684757
862	Herbert Scott, master, clerk (fl.1144x59-1172x78)	0.684217
1022	Odo of Kinninmonth, steward, marischal (d.c.1195)	0.65434
571	Thorald, archdeacon of Lothian (d.1163 or 1166)	0.632719
2978	Stephen, clerk (St Andrews)	0.623444
2781	Abraham of Dunkeld, master, canon (fl.1162x78)	0.621941
493	John of Leicester, bishop of Dunkeld (d.1214)	0.610516
91	Geoffrey, abbot of Dunfermline (d.1178)	0.568139
419	Archibald, abbot of Dunfermline (d.1198)	0.56331
149	Gregory, bishop of Dunkeld (d.1169)	0.561574
1	William I, king of Scots (d.1214)	0.554306
88	William, abbot of Holyrood (I) (d.1172)	0.545819

PART TWO: NETWORK OF ALL LAY (H3) GRANTORS

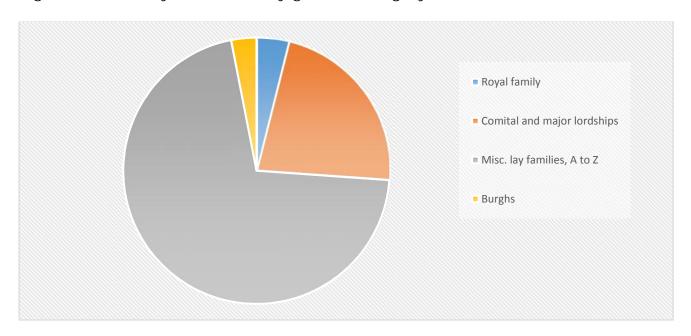
The Social Network Analysis of all lay documents includes 1868 documents, 6893 witnesses, and 66,844 edges. This study is based on the five specified document types in sources with an H-number beginning with 3. Of the 1868 documents, 6 are notifications, one is a charter/brieve, one is an agreement, and 1859 are charters strictly speaking. Table 6.5, below, shows the comparative size of the datasets of royal, ecclesiastical, and lay documents. Despite that the H2 dataset comprised a third fewer documents than the H1 dataset, there are nearly twice as many witnesses in the H2 dataset. Another way of looking at this is that there are three times as many average witnesses per document in the H2 dataset than the H1. This is because there is a much smaller core of royal charter witnesses who appear again and again, while in H2, where there are many different series of charter grantors (e.g. bishops of St Andrews, Glasgow, Aberdeen; abbots of Dunfermline, Melrose, Arbroath, etc.), and thus many more distinct witnesses appear. As far as the lay documents go, the dataset is much larger: there are about twice as many documents as in H1, and about three times as many as in H2. There are also far more distinct witnesses, nearly 6900. This is over three times as many witnesses as in ecclesiastical documents and over six times as many witnesses as in royal documents. With a multitude of different lay grantors and social contexts in H3, it should not be surprising that there are so many individuals appearing as witnesses.

Table 6.5. Comparison of datasets (5 specified document types): royal, ecclesiastical, and private

Н	Grantor category	# docs	# witnesses	# edges	Witness/doc
H1	Royal grantors	935	1039	16,059	1.11
H2	Ecclesiastical grantors	608	2009	19,757	3.3
H3	Lay grantors	1868	6893	66,844	3.69
H4	Two-sided documents	194	1412	13,692	7.28

Of the 1868 documents, 1859 (99.5%) are charters, while only six were notifications, two were charter/brieves, and one was classified as an agreement but was written as a statement in the name of a lay individual (3/327/1). 73 documents (4%) were issued by members of the royal family, 415 (22%) were documents of the comital families and holders of major territorial lordships, such as lords of Galloway, the majority – 1323 or 71% - were charters of a random variety of other lay landholders, and 57 were burgh charters (or three percent).

Figure 6.15. H3 lay documents by grantor category



The three tables below show the top twenty witnesses to H3 documents according to betweenness, degree, and eigenvector centralities. The betweenness centrality list reveals a mix of top lay and church figures. On the lay side, the emphasis is on familiar earls of Fife and Dunbar, as well as justiciars like Walter Oliphant. Perhaps surprisingly, many top church figures also witnessed these lay charters, including the chancellor William del Bois (it is noteworthy that as the person with the highest betweenness in the whole study of all witnesses, William's high betweenness is evident in H1, H2, and H3 studies), as well as bishops of St Andrews, Glasgow, and Moray. The chronological tilt of this dataset is later than those we have examined so far, and Duncan (II) earl of Fife (d.1204) and Jocelin, bishop of Glasgow (d. 1199) are the only twelfth-century figures with high betweenness rankings. Earl Duncan's predominant position in this study helps explain his remarkable, almost unique position in the database as a whole. He holds the top position in terms of all three types of centrality. With a degree of 318, his group of contacts is greater than 60 more than the next most central person. Earl Duncan's connection with so many witnesses to lay documents in addition to royal charter witness, coupled with a respectably high centrality among H2 documents, shows the breadth of social contexts and individuals with whom he had contact.

The large body of texts relating to the holdings of Coldingham priory are very evident in this study, and prominent witnesses from this corpus have been given in italics in the tables below. Four of the twenty most central by degree, and five of the twenty most central by eigenvector are part of this Coldingham group. Alan son of Cospatric of Swinton has an eigenvector score of 94%, which is a

testament to the richness of the Coldingham corpus rather than an indicator of importance on the national stage. It is advisable to consider the Coldingham players and the other actors separately. Further, the study of the H3 documents in the 'Scotland proper dataset' in chapter 7 allows an examination of lay charter witnesses unencumbered by the Coldingham material. Fife and Lothian/Dunbar are two earldoms and provinces which are well-represented in the study. The case study of the acts of the earls of Dunbar, presented later in this chapter, helps explain the significance of Patrick (I), earl of Dunbar (d.1232), William, son of Earl Patrick (I) (d.1253), and Roger de Merlay (II) (d.c.1239). Charters of the Stewart family, earls of Lennox, and others in the area of Glasgow help explain the centrality of two bishops of Glasgow and one archdeacon, as well as the Stewart vassal Robert Crook. Otherwise, many of the central individuals were also top players in the analysis of royal charter witnesses. This includes the highly connected (among royal documents and all documents) group of Earl Duncan (II) of Fife, Philip de Valognes, the chamberlain, and Gilbert earl of Strathearn.

Table 6.6. Top twenty witnesses by betweenness centrality (H3)

Name	Betweeness
Duncan (II), earl of Fife (d.1204)	757128.2
William del Bois, chancellor (d.1232)	740314.2
William Malveisin, bishop of St Andrews (d.1238)	699880.3
Malcolm (I), earl of Fife (d.1229)	562236.2
Patrick (I), earl of Dunbar (d.1232)	511571.5
Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	504928.6
Jocelin, bishop of Glasgow (d.1199)	479103.5
Walter Stewart, earl of Menteith (d.c.1293)	418331
John Hay (I), lord of Naughton (d.xOct.1266)	396605.6
William, son of Earl Patrick (I) (d.1253)	394386.9
Andrew Murray, bishop of Moray (d.1242)	383414.8
Bernard of Hadden, sheriff of Roxburgh	378030.1
David, earl of Huntingdon (d.1219)	370320
Walter Stewart (II), son of Alan (d.1241)	359729.3
Adam of Makerstoun, master, provost (d.1280x86)	358100.6
Roger de Merlay (II) (d.c.1239)	349409.6
John Maxwell, chamberlain, sheriff of Roxburgh (d.1241)	340296.7
Walter of St Albans, bishop of Glasgow (d.1232)	339354.3
Alan, son of Cospatric of Swinton	338024.1
Henry, son of Geoffrey de Liberatione of Perth	336389.2

Table 6.7. Top twenty witnesses by degree centrality (H3)

PoMS ID	Name	Degree
13	Duncan (II), earl of Fife (d.1204)	318
745	Jocelin, bishop of Glasgow (d.1199)	254
42	William del Bois, chancellor (d.1232)	250
1287	Alan, son of Cospatric of Swinton	233
782	Malcolm (I), earl of Fife (d.1229)	226
142	David, earl of Huntingdon (d.1219)	221
40	William Malveisin, bishop of St Andrews (d.1238)	217
1285	Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	204
2115	Walter Lindsay (III), son of William (II) (d.c.1222)	193
444	Patrick (I), earl of Dunbar (d.1232)	192
3149	Patrick, son of Adam son of Aldan the steward	185
880	Bernard of Hadden, sheriff of Roxburgh	182
858	Walter of St Albans, bishop of Glasgow (d.1232)	179
866	Simon, archdeacon of Glasgow (fl.1165x74-1195x96)	179
16019	Robert Crook, knight (12C)	174
15	Philip de Valognes, chamberlain (d.1215)	173
5323	Elias of Prenderguest	172
445	Patrick (II), earl of Dunbar (d.1248)	164
5781	Roger de Merlay (II) (d.c.1239)	163
5423	Henry of Prenderguest (I)	163

Table 6.8. Top twenty witnesses by eigenvector centrality (H3)

Poms ID	Witness	Eigenvector
13	Duncan (II), earl of Fife (d.1204)	1
1287	Alan, son of Cospatric of Swinton	0.942172
142	David, earl of Huntingdon (d.1219)	0.921498
42	William del Bois, chancellor (d.1232)	0.836456
745	Jocelin, bishop of Glasgow (d.1199)	0.835934
782	Malcolm (I), earl of Fife (d.1229)	0.806899
3149	Patrick, son of Adam son of Aldan the steward	0.775714
2115	Walter Lindsay (III), son of William (II) (d.c.1222)	0.770019
40	William Malveisin, bishop of St Andrews (d.1238)	0.767788
444	Patrick (I), earl of Dunbar (d.1232)	0.765531
1285	Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	0.718511
5423	Henry of Prenderguest (I)	0.717066
858	Walter of St Albans, bishop of Glasgow (d.1232)	0.702635
15	Philip de Valognes, chamberlain (d.1215)	0.697596
5323	Elias of Prenderguest	0.686796
445	Patrick (II), earl of Dunbar (d.1248)	0.665752
798	Richard de Prebenda, bishop of Dunkeld (d.1210)	0.649437
5312	Adam of Little Reston	0.640545
6177	Elias of Ayton (father of John)	0.626034
260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	0.623787

Figure 6.16. Netdraw: witnesses to H3 documents

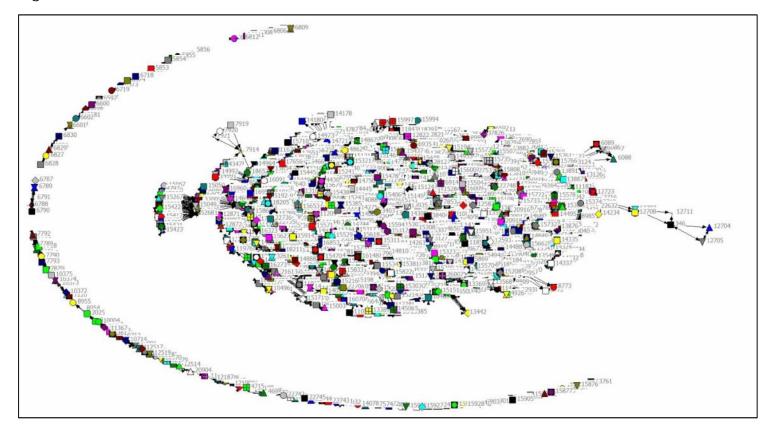


Figure 6.17. Netdraw: witnesses to H3 documents, >3 co-witnessing instances

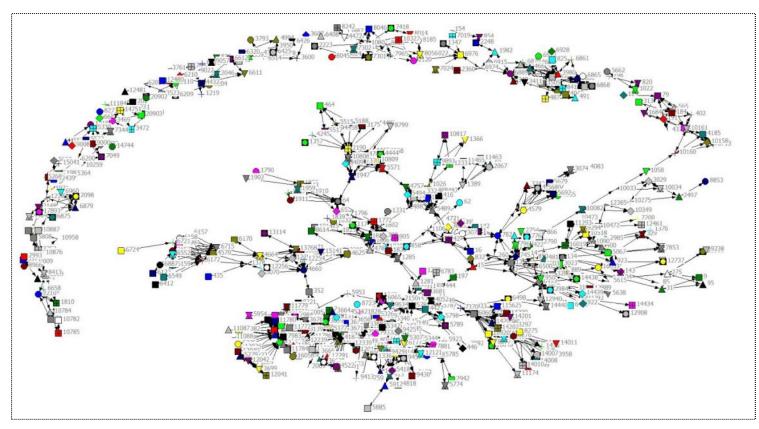


Figure 6.18. Netdraw: witnesses to H3 documents, >5 co-witnessing instances

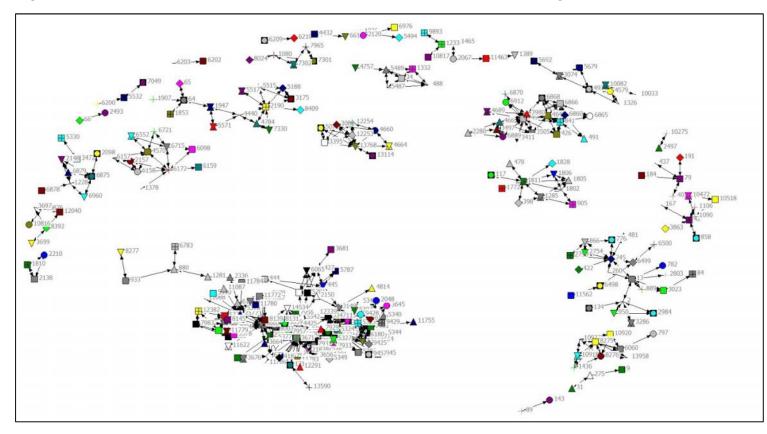


Figure 6.19. Netdraw: witnesses to H3 documents, >8 co-witnessing instances

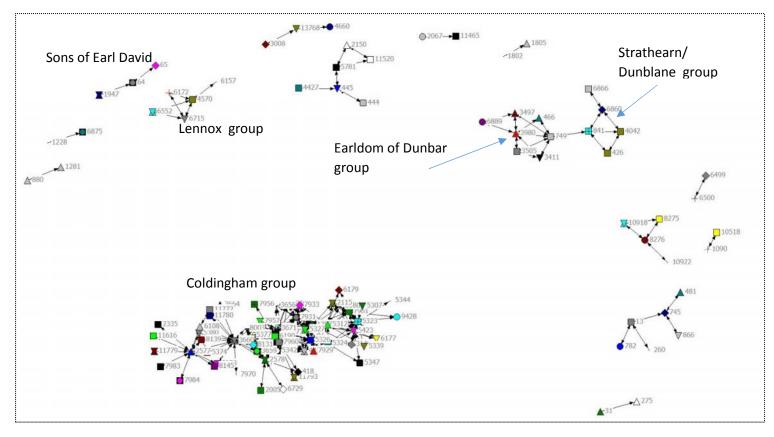


Figure 6.20. Netdraw: witnesses to H3 documents, >10 co-witnessing instances

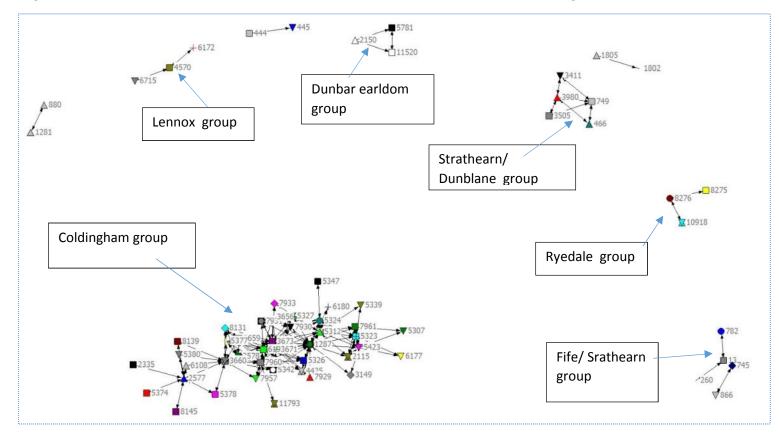


Figure 6.21. Netdraw: witnesses to H3 documents, >13 co-witnessing instances

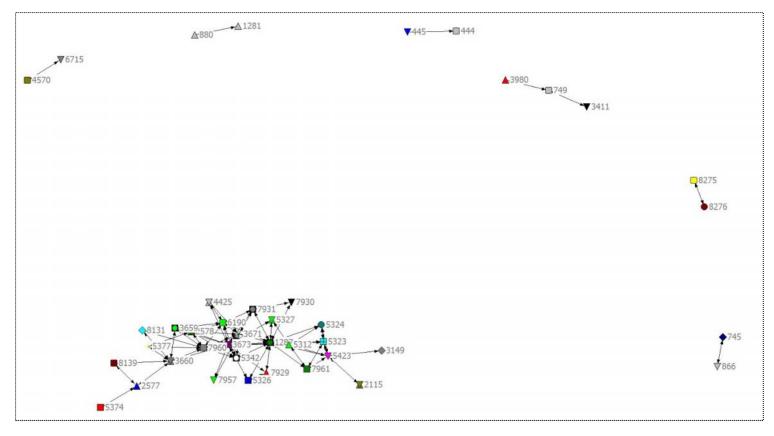


Figure 6.22. Netdraw: witnesses to H3 documents, >15 co-witnessing instances

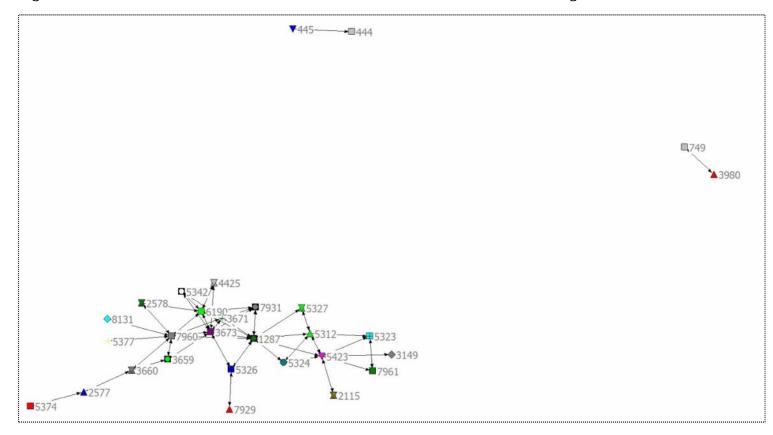


Figure 6.23. Netdraw: witnesses to H3 documents, >18 co-witnessing instances

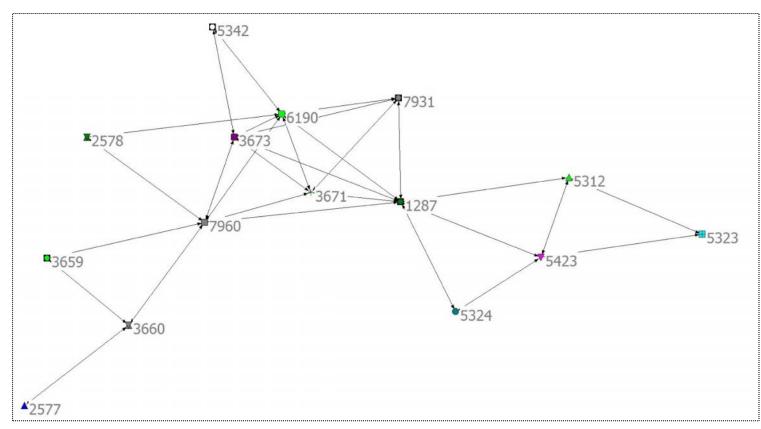


Figure 6.24. Netdraw: witnesses to H3 documents, >20 co-witnessing instances

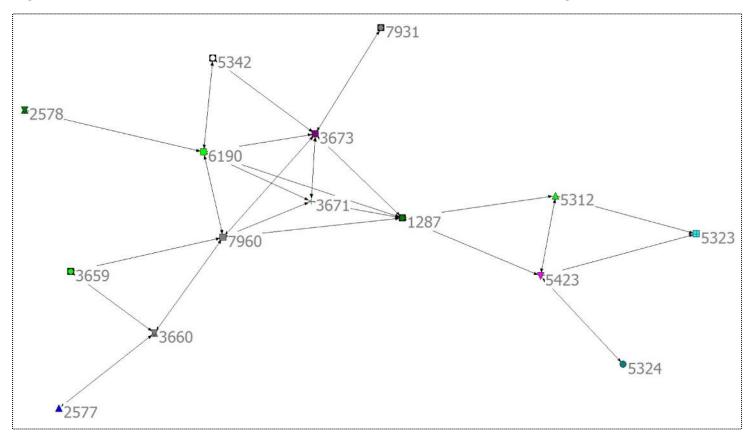


Figure 6.25. Netdraw: witnesses to H3 documents, >23 co-witnessing instances

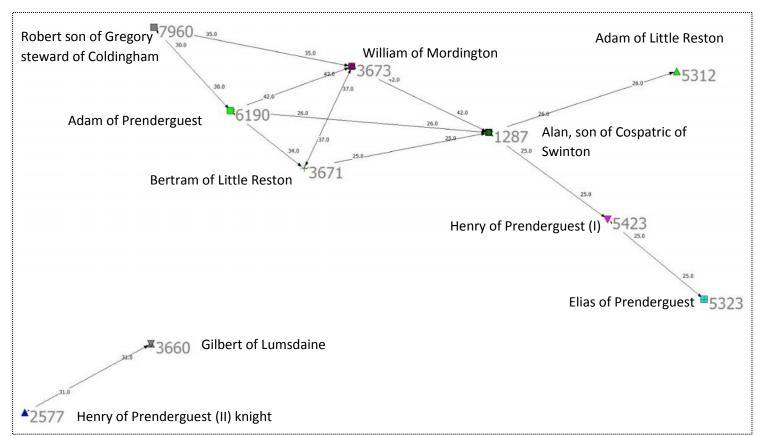


Figure 6.26. Netdraw: witnesses to H3 documents, >25 co-witnessing instances

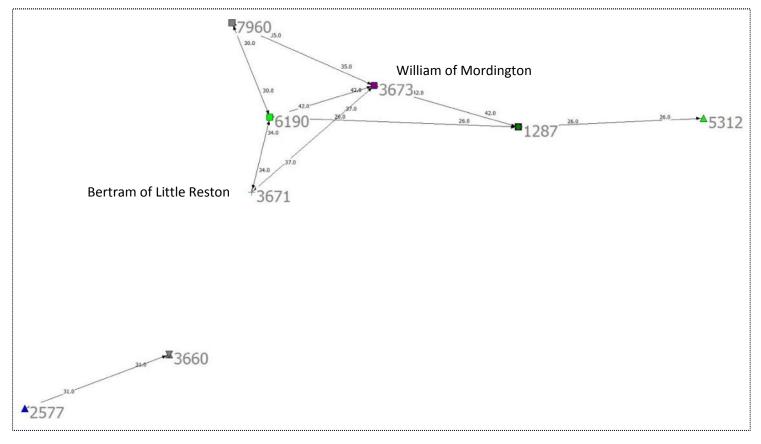
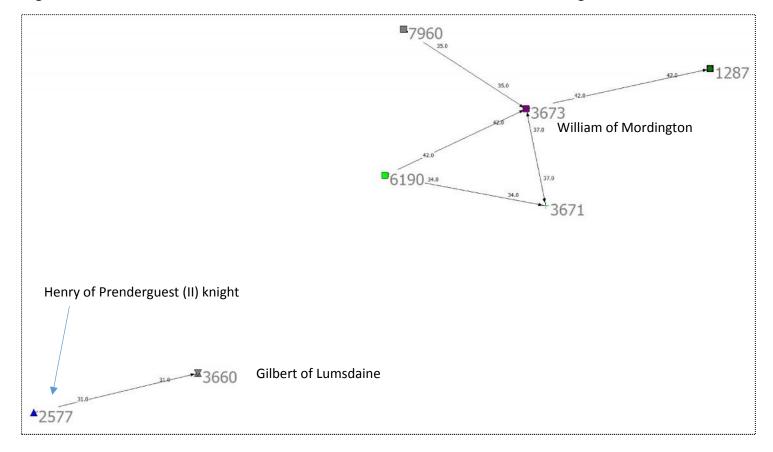


Figure 6.27. Netdraw: witnesses to H3 documents, >30 co-witnessing instances



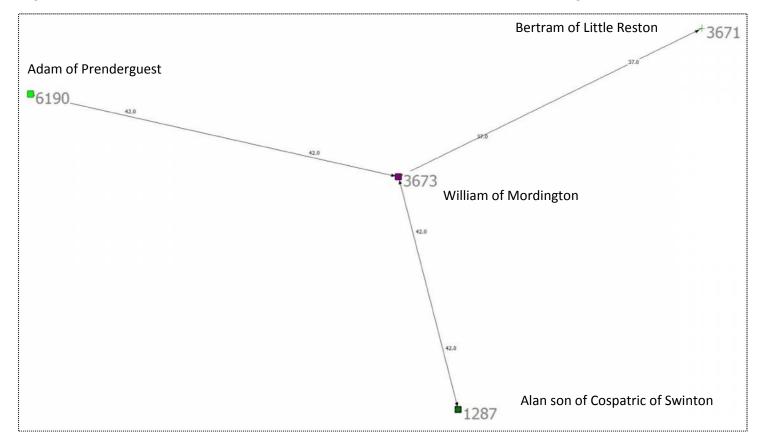


Figure 6.28. Netdraw: witnesses to H3 documents, >35 co-witnessing instances

Figure 6.19 reveals a few of the social contexts in which witnesses appeared, at the level of witnessing together more than eight times. The Coldingham group at that level is joined by smaller groups centering on Lennox, Strathearn/ Dunblane, Dunbar, Fife & Strathearn, and the Ryedale family. The Coldingham witnesses dominate the higher reaches of co-witnessing acts, however, as tables 6.24 through 6.28 show. Individuals and families based on Prenderguest, Swinton, Mordington, Reston, Ayton and Lumsdaine are all evident. Table 6.9 shows that every co-witnessing 'relationship' involving the witnessing of twenty or more documents involved individuals from the Coldingham group. William of Mordington in particular emerges as a key figure: he was a part of the top four pairs.

Table 6.9. Most productive co-witnessing 'relationships' (H3)

Person 1	Person 2	# docs
Alan, son of Cospatric of Swinton	William of Mordington	42
William of Mordington	Adam of Prenderguest	42
William of Mordington	Bertram of Little Reston (son of Adam of Little Reston)	37
William of Mordington	Robert, son of Gregory steward of Coldingham	35
Adam of Prenderguest	Bertram of Little Reston (son of Adam of Little Reston)	34
Gilbert of Lumsdaine	Henry of Prenderguest (II) knight	31
Adam of Prenderguest	Robert, son of Gregory steward of Coldingham	30
Alan, son of Cospatric of Swinton	Adam of Little Reston	26
Alan, son of Cospatric of Swinton	Adam of Prenderguest	26
Henry of Prenderguest (I)	Alan, son of Cospatric of Swinton	25
Henry of Prenderguest (I)	Elias of Prenderguest	25
Alan, son of Cospatric of Swinton	Bertram of Little Reston (son of Adam of Little Reston)	25
Alan, son of Cospatric of Swinton	Robert, son of Gregory steward of Coldingham	23
William of Scremerston, knight	Adam of Prenderguest	23
Robert, son of Gregory steward of Coldingham	Bertram of Little Reston (son of Adam of Little Reston)	23
Gilbert of Lumsdaine	David of Lumsdaine	23
Henry of Prenderguest (I)	William of Lumsdaine	22
William of Mordington	John, son of Elias of Ayton	22
Henry of Prenderguest (I)	Adam of Little Reston	21
Thomas of Nisbet	William of Mordington	21
Thomas of Nisbet	Adam of Prenderguest	21
Adam of Little Reston	Elias of Prenderguest	21
Robert, son of Gregory steward of Coldingham	Gilbert of Lumsdaine	21
Robert, son of Gregory steward of Coldingham	David of Lumsdaine	21
Alan, son of Cospatric of Swinton	John, son of Elias of Ayton	20
Adam of Prenderguest	John, son of Elias of Ayton	20
John, son of Elias of Ayton	Bertram of Little Reston (son of Adam of Little Reston)	20

PART THREE: NETWORK OF ALL H4 DOCUMENTS

There are 194 documents in the dataset, which comprises mostly two-sided documents recording agreements and settlements. There were 1412 witnesses, of whom only one (Queen Ermengarde) was a woman. There were 13,692 edges in the study. Due to the relatively small number of documents in the dataset and the tendency for agreements to record a critical mass of legal witnesses, there are an average of over 7 witnesses per document in this study.

The main characteristic of the H4 dataset is that is a collection of highly disparate examples, with not a great degree of overlap. Few individuals witnessed together more than even once or twice. There is a case to be made that many of these agreements should be included in other case studies involving lay or ecclesiastical grantors, but that would involve painstaking crafting of bespoke datasets, so will have to be left for some time in the future. Interestingly, David earl of Huntingdon (d. 1219), brother of King William, had the highest degree and eigenvector centrality, although the majority of top players were bishops, abbots, archdeacons and other clergy. Indeed Earl David and King William himself were the only laymen to appear in the top 14 actors according to degree centrality, and in the top 19 according to eigenvector.

Table 6.10. Top twenty witnesses by betweenness centrality (H4)

Poms ID	Name	Betweeness
59	Gregory, bishop of Brechin (fl.1189x98-1242x46)	57453.19
1	William I, king of Scots (d.1214)	52716.99
445	Patrick (II), earl of Dunbar (d.1248)	51586.4
474	Guy, abbot of Lindores (d.1219)	48093.68
2047	Peter Ramsay, bishop of Aberdeen (d.1256)	44245.61
2115	Walter Lindsay (III), son of William (II) (d.c.1222)	42597.28
900	Adam, son of Gilbert, son of Richer, lord of Kilbucho	42446
4743	Fearchar, earl of Ross (d.1251)	39252.57
485	Jocelin, archdeacon of Dunkeld (fl.1165x67-1193/4)	35881.04
42	William del Bois, chancellor (d.1232)	35398.16
142	David, earl of Huntingdon (d.1219)	33617.82
789	Gilbert Murray, bishop of Caithness (d.1245)	32936.83
757	Richard of Coldingham, master (d.1198)	32846.94
2491	Stephen of Lilliesleaf, master, clerk, persona	32533.85
5330	Henry, son of Geoffrey de Liberatione of Perth	30456
835	Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	27899.86
760	Brice Douglas, bishop of Moray (d.1222)	27242.42
858	Walter of St Albans, bishop of Glasgow (d.1232)	26530.04
2110	William of Brechin, knight	26036
768	William of Greenlaw, master (d.1247)	26003.83

Table 6.11. Top twenty witnesses by degree centrality (H4)

Poms ID	Name	Degree
142	David, earl of Huntingdon (d.1219)	124
2491	Stephen of Lilliesleaf, master, clerk, persona	122
863	Isaac Scott, master, clerk	117
474	Guy, abbot of Lindores (d.1219)	111
1	William I, king of Scots (d.1214)	106
42	William del Bois, chancellor (d.1232)	105
850	John Scott, bishop of Dunkeld (d.1203)	101
829	Ranulf de Wat, archdeacon of St Andrews (d.1209)	97
835	Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	96
493	John of Leicester, bishop of Dunkeld (d.1214)	95
858	Walter of St Albans, bishop of Glasgow (d.1232)	93
809	Ralph, bishop of Brechin (d.1212x14)	86
40	William Malveisin, bishop of St Andrews (d.1238)	85
39	Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	84
445	Patrick (II), earl of Dunbar (d.1248)	82
4743	Fearchar, earl of Ross (d.1251)	82
798	Richard de Prebenda, bishop of Dunkeld (d.1210)	82
500	Osbert, abbot of Kelso (d.1203)	81
794	William, abbot of Holyrood (II) (fl.1187x89-1206)	78
2115	Walter Lindsay (III), son of William (II) (d.c.1222)	77

Table 6.12. Top twenty witnesses by eigenvector centrality (H4)

Poms ID	Name	Eigenvector
142	David, earl of Huntingdon (d.1219)	1
1	William I, king of Scots (d.1214)	0.908997
850	John Scott, bishop of Dunkeld (d.1203)	0.846207
829	Ranulf de Wat, archdeacon of St Andrews (d.1209)	0.83654
809	Ralph, bishop of Brechin (d.1212x14)	0.827825
798	Richard de Prebenda, bishop of Dunkeld (d.1210)	0.815957
863	Isaac Scott, master, clerk	0.80627
39	Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	0.792588
794	William, abbot of Holyrood (II) (fl.1187x89-1206)	0.788068
770	William of Hailes, master, dean of St Andrews (fl.1189x98)	0.775541
3057	Ralph, clerk of Bishop Roger of St Andrews	0.775541
820	Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	0.764114
500	Osbert, abbot of Kelso (d.1203)	0.759679
493	John of Leicester, bishop of Dunkeld (d.1214)	0.73906
474	Guy, abbot of Lindores (d.1219)	0.727717
858	Walter of St Albans, bishop of Glasgow (d.1232)	0.718067
42	William del Bois, chancellor (d.1232)	0.713415
40	William Malveisin, bishop of St Andrews (d.1238)	0.656168
1450	Hugh, king's chaplain (TRW)	0.609706
260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	0.601789

Figure 6.29. Netdraw: witnesses to H4 documents, >1 co-witnessing instances

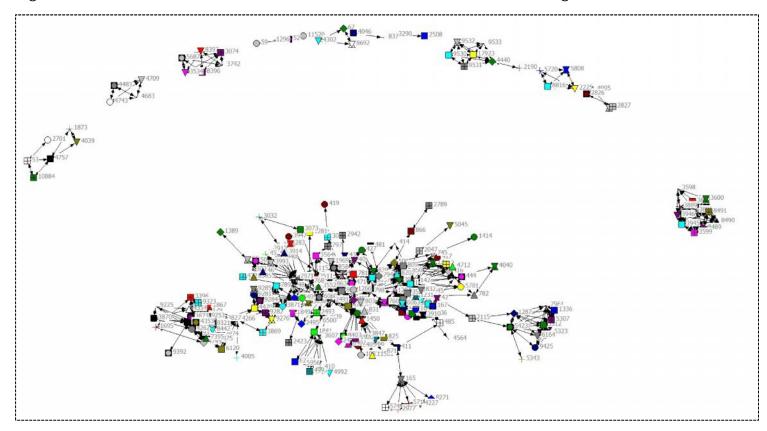


Figure 6.29. Netdraw: witnesses to H4 documents, >2 co-witnessing instances

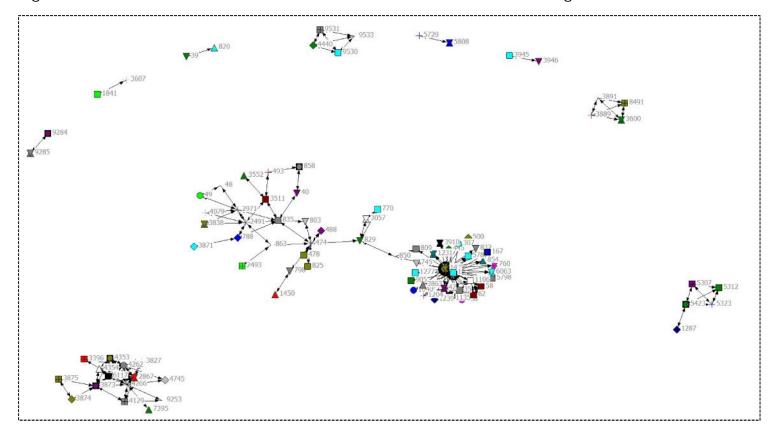


Figure 6.29. Netdraw: witnesses to H4 documents, >3 co-witnessing instances

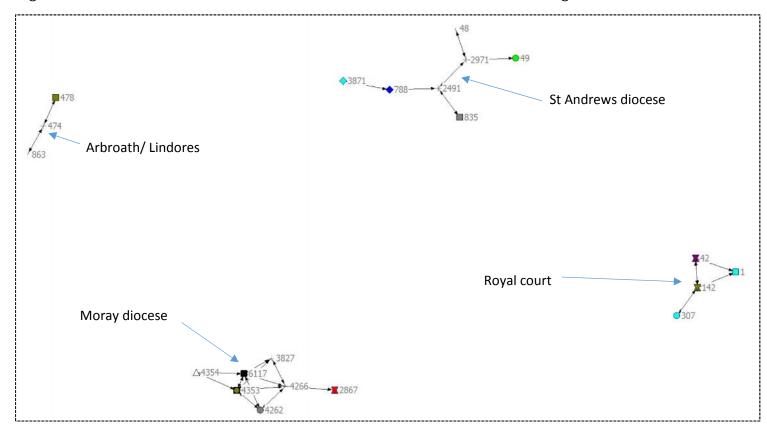


Figure 6.29. Netdraw: witnesses to H4 documents, >4 co-witnessing instances

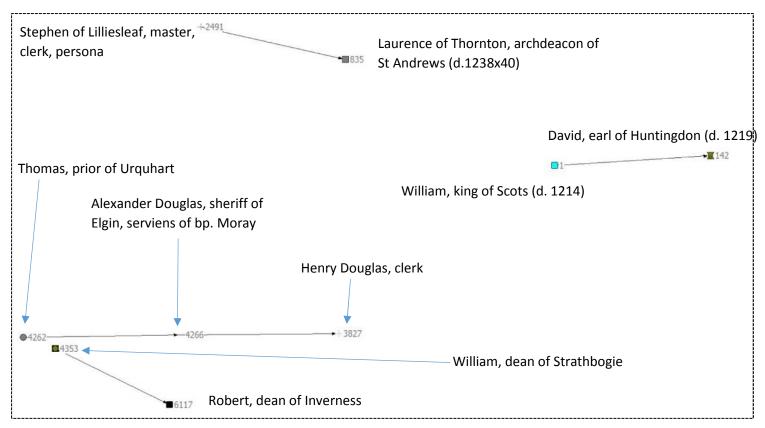


Table 6.13 highlights the importance of the diocese of Moray in the body of two-sided documents. The bishops of Moray had a marked preference for making formal agreements with magnates and others in the north of Scotland. This explains the predominance of individuals who co-witnessed relatively more than others in H4, despite the very low level of co-witnessing across the board. These men were Alexander Douglas, sheriff of Elgin, serviens of bp. Moray; Henry Douglas, clerk; Thomas, prior of Urquhart (fl.1226-32); William, dean of Strathbogie; Robert, dean of Inverness; Andrew Murray, bishop of Moray (d.1242); Augustine of Elgin and Gregory, dean of Strathspey.

Table 6.13. Most productive co-witnessing 'relationships' (H4)

Person 1	Person 2	#docs
David, earl of Huntingdon (d.1219)	William I, king of Scots (d.1214)	8
Laurence of Thornton, archdeacon of St Andrews (d.1238×40)	Stephen of Lilliesleaf, master, clerk, persona	7
Alexander Douglas, sheriff of Elgin, serviens of bp. Moray	Henry Douglas, clerk	5
Alexander Douglas, sheriff of Elgin, serviens of bp. Moray	Thomas, prior of Urquhart (fl.1226-32)	5
Robert, dean of Inverness	William, dean of Strathbogie	5
Alexander Douglas, sheriff of Elgin, serviens of bp. Moray	Augustine of Elgin	4
Alexander Douglas, sheriff of Elgin, serviens of bp. Moray	Robert, dean of Inverness	4
Alexander Douglas, sheriff of Elgin, serviens of bp. Moray	William, dean of Strathbogie	4
Andrew Murray, bishop of Moray (d.1242)	Edward Murray, master, canon, bishop's clerk	4
Andrew Murray, bishop of Moray (d.1242)	Stephen of Lilliesleaf, master, clerk, persona	4
David, earl of Huntingdon (d.1219)	Robert of London (d.1225)	4
David, earl of Huntingdon (d.1219)	William del Bois, chancellor (d.1232)	4
Gregory, dean of Strathspey	Robert, dean of Inverness	4
Gregory, dean of Strathspey	William, dean of Strathbogie	4
Guy, abbot of Lindores (d.1219)	Henry, abbot of Arbroath (fl.1179-1207)	4
Guy, abbot of Lindores (d.1219)	Isaac Scott, master, clerk	4
Henry Douglas, clerk	Robert, dean of Inverness	4
Henry Douglas, clerk	William, dean of Strathbogie	4
Peter, chaplain and clerk of Bishop Malveisin	Simon de Noisy, clerk of Bishop William of St Andrews	4
Peter, chaplain and clerk of Bishop Malveisin	Stephen of Lilliesleaf, master, clerk, persona	4
Peter, chaplain and clerk of Bishop Malveisin	William of Gullane, rector of Gullane	4
Robert, dean of Inverness	Thomas, prior of Urquhart (fl.1226-32)	4
Thomas, prior of Urquhart (fl.1226-32)	William, dean of Strathbogie	4
William del Bois, chancellor (d.1232)	William I, king of Scots (d.1214)	4

PART FOUR: GRANTOR CASE STUDIES

1. Documents of the bishops of St Andrews (H2/10/)

There were 194 documents, of which 192 were charters and two were notifications. There were 600 witnesses, of whom only one was a woman, and 8306 edges in the study.

The most central individuals in the network were active in the mid-to-late twelfth century. The Gephi soiciogram below shows the especially well-represented grouping at this chronological period. Matthew, bishop of Aberdeen, the most central person by both degree and eigenvector, owes his prominence primarily to his position as archdeacon of St Andrews from around 1150 to 1172. His successor in the office, Walter of Roxburgh, archdeacon from 1173 to some point between 1179 and 1188, was the fourteenth most central person by degree. Ranulf de Wat, who was archdeacon from 1199 to 1209, had a degree almost as high as Matthew, but had a much lower eigenvector – only about 67%. Matthew's Lothian counterpart, Andrew, had the second highest eigenvector score, at 97.5%. The archdeacon of Lothian, Thorald, also appears in our league tables. John of Leicester, bishop of Dunkeld from 1212 to 1214, appears here in the 'top ten' because he was archdeacon of Lothian from 1200 to 1212. John Scott, bishop of Dunkeld from around 1183 to 1203, had been a member of the St Andrews familia, was a magister (equivalent of university graduate), and was the St Andrews' church establishment's choice for bishop in 1178 against the wishes of the king. Most of the other central figures listed below were members of the bishops' familia in the later twelfth century, as clerks, chaplains, chancellors, and so on. Most of these men were clerics, also we should also note the steward Odo of Kinninmonth and the doorward Gamel.

Table 6.14. Centrality by Degree

Poms ID	Name	Eigenvector	Degree
2	Matthew, bishop of Aberdeen (d.1199)	1	170
829	Ranulf de Wat, archdeacon of St Andrews (d.1209)	0.666611	169
411	Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	0.975104	167
3016	Alexander, chaplain of bishops of St Andrews (12C)	0.838007	165
165	Aiulf, dean of Lothian (fl.1150/51-1186)	0.967106	163
862	862 Herbert Scott, master, clerk (fl.1144x59-1172x78)		145
850	John Scott, bishop of Dunkeld (d.1203)	0.819292	143
571	Thorald, archdeacon of Lothian (d.1163 or 1166)	0.830259	140
493	John of Leicester, bishop of Dunkeld (d.1214)	0.57092	128

271	Robert, son of Saewulf, bishop's chancellor	0.800845	122
202	202 Andrew, bishop of Caithness (d.1184)		117
13	Duncan (II), earl of Fife (d.1204)	0.779377	111
2781	2781 Abraham of Dunkeld, master, canon (fl.1162x78)		111
821	Walter of Roxburgh, archdeacon of St Andrews (fl.1165x72-1179x88)	0.552546	106
863	Isaac Scott, master, clerk	0.442392	105
1022	Odo of Kinninmonth, steward, marischal (d.c.1195)	0.691995	104
2483	2483 Gamel, doorward, master (St Andrews)		104
3072	3072 Richard, chaplain of Bishop Roger of St Andrews		104
2978	Stephen, clerk (St Andrews)	0.764677	103
133	Nicholas of Roxburgh, chancellor (d.1171?)	0.70273	99

The table of people who witnessed alongside one another most frequently is likewise a testament to the richness of the mid-to-late twelfth century charter material, the longevity of many of the careers, and, perhaps, the cohesiveness of the relationships formed between these men. The positions of some of these men, such as those who served as archdeacons, should not surprise us, but the appearance of some other witnesses is perhaps less expected. The chaplain Alexander witnessed 63 surviving document texts, nearly all of them episcopal charters and agreements of the bishops. His career spanned thirty years from the early 1160s to (most likely) the early 1190s. Three of the top five cowitnessing 'relationships' involve Alexander the chaplain. Men like these provided important institutional continuity through changes of personnel higher up the food chain. Master Herbert Scot was also an important figure in the formative period of the 1150s and 1160s. He witnessed 29 times with Matthew, who was archdeacon of St Andrews at the time, and 27 times with Andrew, archdeacon of Lothian. We know from historical evidence that Matthew was at the centre of a 'network' derived which represented the legacy of Bishop Robert (1124-59). Master Herbert Scot, Master John Scot, and Aiulf dean of Lothian were key members of this power grouping (Hammond, 'Founding of the Burgh', 80): all four of these men are in the top 7 by degree and have eigenvector scores over 80%. Andrew, archdeacon of Lothian, however, appears to have acted in opposition to them (Watt, Graduates). Ten of the most productive co-witnessing 'relationships' include one or two of these four men. Towards the lower end of the following table, we find a few personnel of the time of Bishop William Malveisin (1202-38), most notably the archdeacon of St Andrews, Laurence of Thornton and the bishop's chaplain and clerk, Peter.

Table 6.15. Most productive co-witnessing 'relationships' (H2/10)

Person 1	Person 2	# docs
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Alexander, chaplain of bishops of St Andrews (12C)	32
Alexander, chaplain of bishops of St Andrews (12C)	Walter of Roxburgh, archdeacon of St Andrews (fl.1165x72-1179x88)	30
Matthew, bishop of Aberdeen (d.1199)	Herbert Scott, master, clerk (fl.1144x59-1172x78)	29
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Herbert Scott, master, clerk (fl.1144x59-1172x78)	27
Alexander, chaplain of bishops of St Andrews (12C)	Abraham of Dunkeld, master, canon (fl.1162x78)	27
Aiulf, dean of Lothian (fl.1150/51-1186)	Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	26
Aiulf, dean of Lothian (fl.1150/51-1186)	Herbert Scott, master, clerk (fl.1144x59-1172x78)	23
Robert, son of Saewulf, bishop's chancellor	Alexander, chaplain of bishops of St Andrews (12C)	23
Matthew, bishop of Aberdeen (d.1199)	Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	21
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Abraham of Dunkeld, master, canon (fl.1162x78)	21
John Scott, bishop of Dunkeld (d.1203)	Alexander, chaplain of bishops of St Andrews (12C)	21
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Peter, chaplain and clerk of Bishop Malveisin	21
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Edward Murray, master, canon, bishop's clerk	21
Andrew, archdeacon of Lothian (fl.1147x59-1178x84)	Robert, son of Saewulf, bishop's chancellor	20
Herbert Scott, master, clerk (fl.1144x59-1172x78)	Robert, son of Saewulf, bishop's chancellor	20
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Michael, master, clerk, chaplain (fl.1201-1220x25)	20
Peter, chaplain and clerk of Bishop Malveisin	Stephen of Lilliesleaf, master, clerk, persona	20
Peter, chaplain and clerk of Bishop Malveisin	Michael, master, clerk, chaplain (fl.1201-1220x25)	20

Figure 6.29. Gephi: witnesses to documents of the bishops of St Andrews (H2/10)

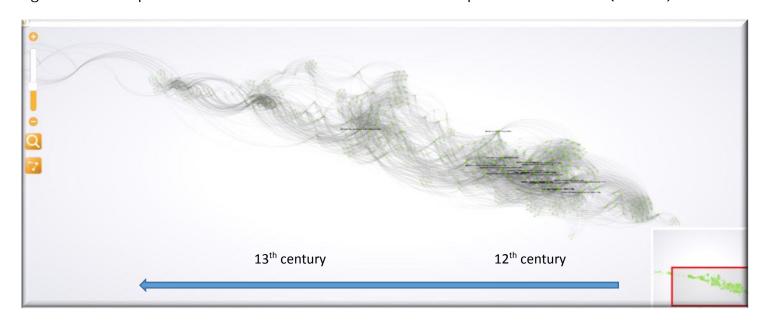


Figure 6.30. Gephi: witnesses to H2/10 documents, close-up

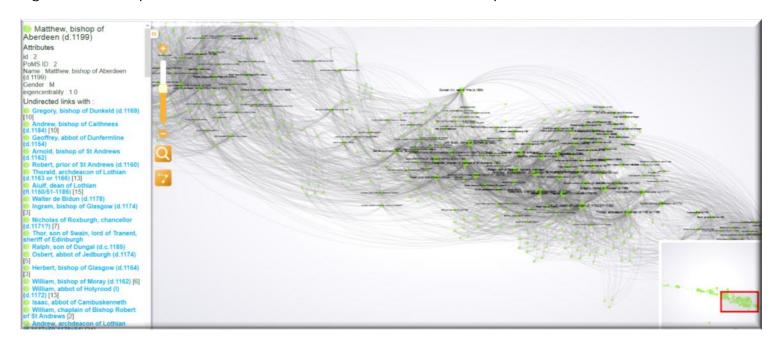


Figure 6.31. Gephi: witnesses to H2/10 documents, close-up

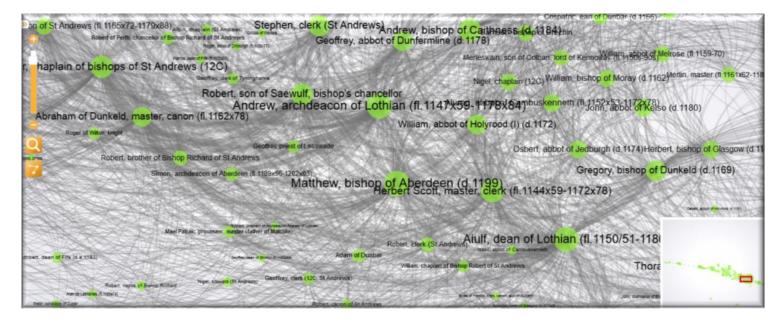


Figure 6.32. Netdraw: witnesses to H2/10 documents

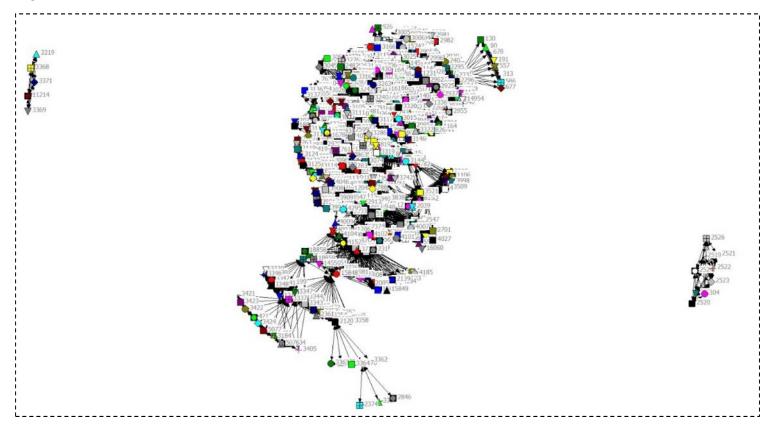
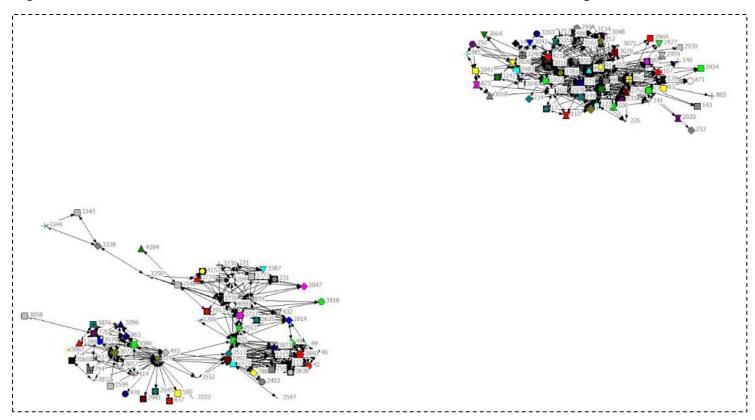


Figure 6.33. Netdraw: witnesses to H2/10 documents, >3 co-witnessing instances



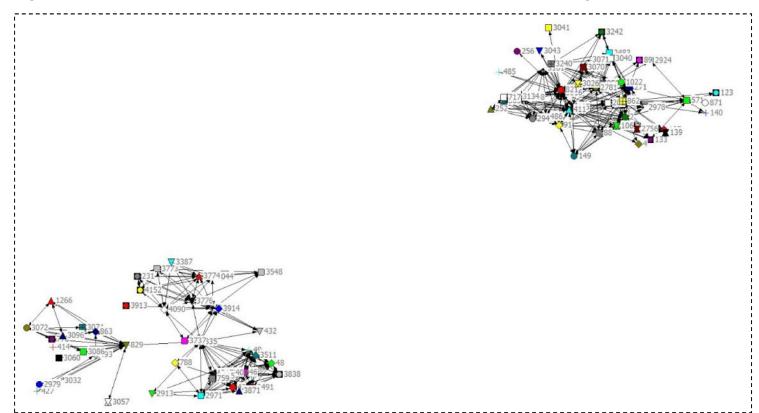


Figure 6.34. Netdraw: witnesses to H2/10 documents, >5 co-witnessing instances

At the level of more than three witnessing connections, it is clear that we have two main segments. The segment in the top right is highly interconnected and represents the group of individuals active in the second half of the twelfth century who have the highest centrality and were discussed above. The segment in the bottom left is bifurcated, with only the link between numbers 829 and 3737 connecting the two 'halves'. Number 829 is the archdeacon Ranulf de Wat (d. 1209) and number 835, his successor as archdeacon of St Andrews, Laurence of Thornton (d. 1240). Thus we have two completely disconnected groups of St Andrews personnel, one from the mid-to-late twelfth century, the other from the early-to-mid thirteenth century.

Figure 6.35. Netdraw: witnesses to H2/10 documents, >8 co-witnessing instances

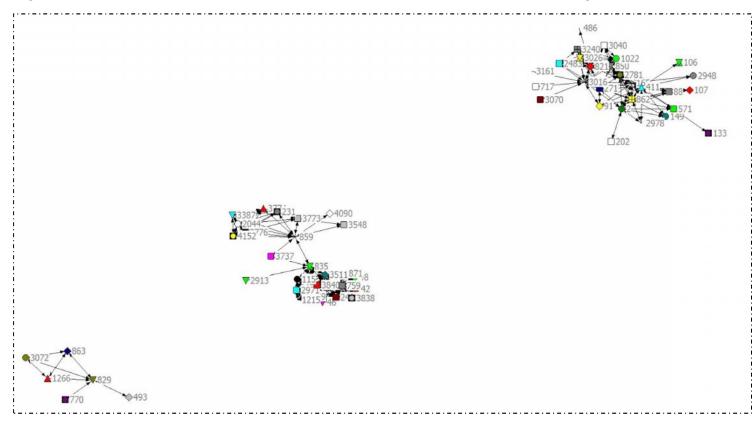
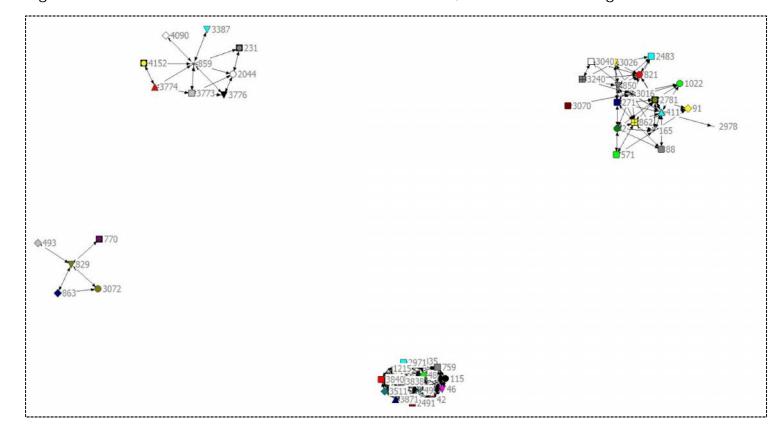


Figure 6.36. Netdraw: witnesses to H2/10 documents, >10 co-witnessing instances



At the level of more than ten co-witnessing instances, we are left with four distinct and separate segments. On the left we have a group of five, centred around no 829, archdeacon Ranulf de Wat (d. 1209). These men are John of Leicester [493], his counterpart archdeacon in Lothian, Master William of Hailes, dean of St Andrews [770], a 'dean of Christianity', Bishop Roger's chaplain, Richard [3072], and the clerk Master Isaac Scot [863]. Moving clockwise, there is another segment focused around the central figure of Master Alexander de St Martin [859], who was active from the 1220s through 1240s, and was the archdeacon of Lothian's official at one point. He is involved in five triads and one clique of four. These men include the masters Alexander of Edinburgh [3773] and Hugh of Melbourne [3776], and Adam, archdeacon of Lothian in the 1240s [231]. The larger, more interconnected segment in the top right, represents the twelfth-century group we have already encountered, including [2] Matthew, bishop of Aberdeen (d. 1199), [862] Master Herbert Scot, [271] the episcopal chancellor Robert son of Saewulf, but also including contemporaneous abbots, e.g. [88] William (I), abbot of Holyrood (d. 1172) and [91] Geoffrey, abbot of Dunfermline (d. 1178). The segment at the bottom of the graph represents the personnel of the episcopate of William Malveisin (1202-38), and includes [3840] Peter of Dryburgh, master, clerk, [3838] Adam Ovid, master (fl.1203-33), [3871] Edward Murray, master, canon, bishop's clerk, [46] Richard of Dover (Tyninghame), master, clerk, [48] Simon de Noisy, clerk of Bishop William of St Andrews, and [49] William of Gullane, rector of Gullane. It also includes the chancellor William del Bois [42], whom we have encountered on multiple occasions. This is more visible on the sociogram of more than 13 co-witnessing acts, below.

Figure 6.37. Netdraw: witnesses to H2/10 documents, >13 co-witnessing instances

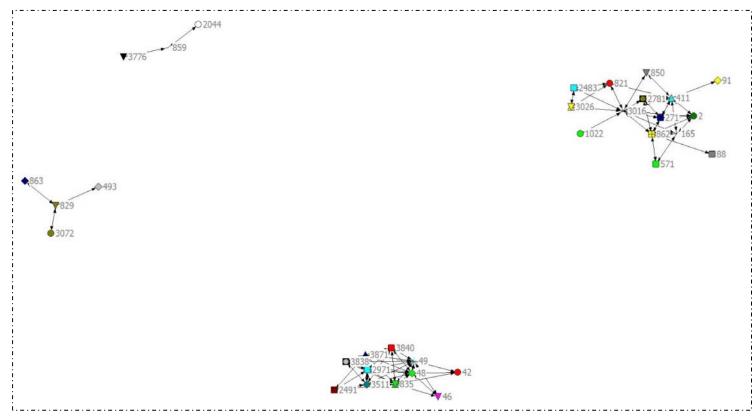


Figure 6.38. Netdraw: witnesses to H2/10 documents, >15 co-witnessing instances

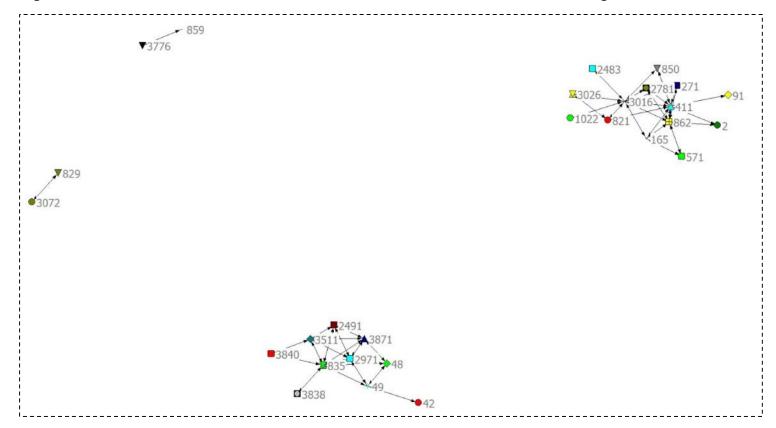


Figure 6.39. Netdraw: witnesses to H2/10 documents, >17 co-witnessing instances

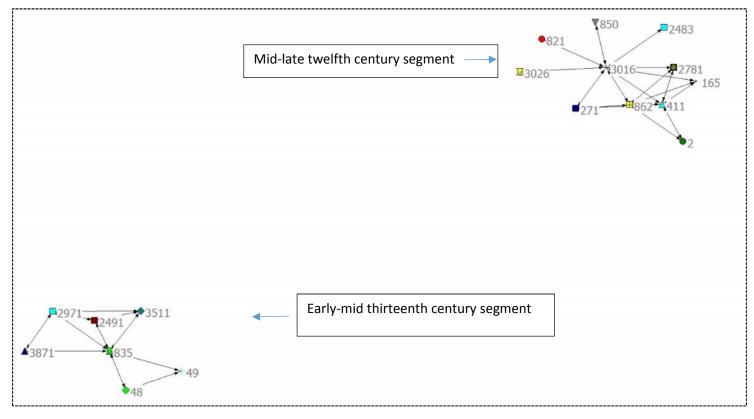


Figure 6.40. Netdraw: witnesses to H2/10 documents, >20 co-witnessing instances

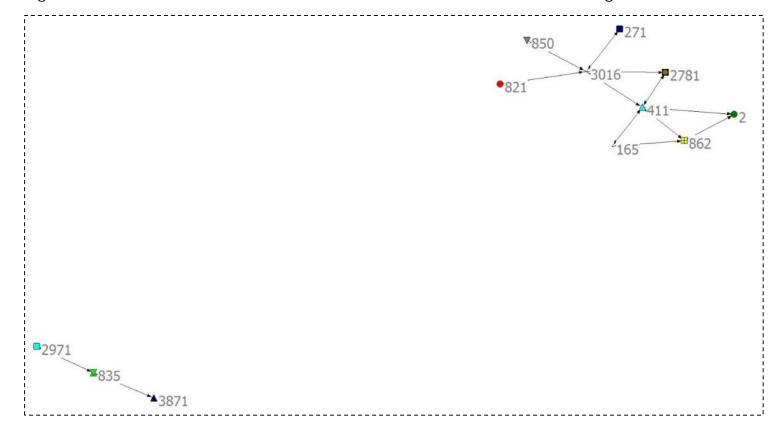


Figure 6.41. Netdraw: witnesses to H2/10 documents, >25 co-witnessing instances

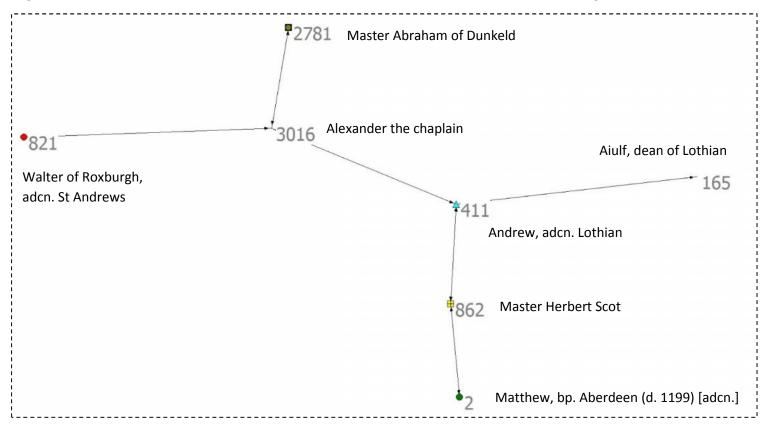
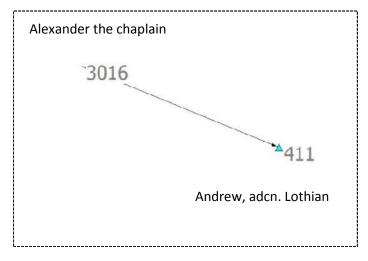


Figure 6.42. Netdraw: witnesses to H2/10 documents, >30 co-witnessing instances



References

Hammond, Matthew (2015), 'The bishop, the prior, and the founding of the burgh of St Andrews', Innes Review 66:1, pp. 72-101.

D. E. R. Watt (1977), A Biographical Dictionary of Scottish Graduates to A.D. 1410. Oxford.

2. Documents of the bishops of Glasgow (H2/7)

There were 52 documents, 50 of which were charters and two of which were notifications. There were 242 witnesses, all of them male. The network had 2147 edges.

In parallel with the charters of the bishops of St Andrews, the period in the documents of the bishops of Glasgow which jumps out in Social Network Analysis is in the mid-to-late twelfth century, although while in St Andrews the 1150s and 1160s, including the reigns of Arnold (1160-62) and Richard (1163-78) were to the fore, in Glasgow the episcopacies of Ingram (1164-74) and especially Jocelin (1175-99) seem most relevant. As with St Andrews, archdeacons, officials, deans of Christianity and other clerks and canons are among the most central players. The archdeacon Simon was in office for most of the time of both bishops Ingram and Jocelin, although while he has the highest number of co-witnessing contacts (85), it is his younger contemporary Herbert, the dean of the cathedral chapter of Glasgow from around 1180 to the 1200s, who has the highest eigenvector score. The counterpart of Alexander the chaplain at St Andrews, in terms of being a long-serving and apparently very well-connected churchman whose lack of any other title might cause him to be overlooked, is the clerk Walter, who served from the early 1170s to around 1195, and had an eigenvector of 98.5% that of Dean Herbert's.

Table 6.17. Centrality: top 15 by degree

PoMS ID	Name	Degree	Eigenvector Centrality
866	Simon, archdeacon of Glasgow (fl.1165x74-1195x96)	85	0.988227
776	John of Huntingdon, master, official of Glasgow (fl.1179x1208)	84	0.99228
2754	Walter, clerk of Bishops Ingram and Jocelin	83	0.985137
926	Elias of Partick, clerk, canon (son of Fulbert)	79	0.95064
481	Herbert, dean of Glasgow (fl.1179x89-1204x07)	77	1
1725	Bede, canon of Glasgow	73	0.975962
1718	William, canon of Glasgow, clerk	55	0.840206
2802	Peter, dean of Clydesdale and Stobo (fl.1175x95)	53	0.843521
797	Robert, archdeacon of Glasgow (d.1222)	51	0.340527
1738	Elias, canon of Glasgow (2)	47	0.733039
1124	John of Roxburgh, master, treasurer of Glasgow (d.1196)	46	0.726274
2895	John, dean of Teviotdale and Roxburgh (fl.1195-1204x07)	46	0.720881
42	William del Bois, chancellor (d.1232)	41	0.6955
774	Hugh, abbot of Newbattle (fl.1179-1201)	41	0.705128
2910	Gervase, clerk (king's and Glasgow)	41	0.711421

The most productive 'relationships' of individuals based on their co-witnessing can be viewed below. The most significant is that between Simon the archdeacon and Walter the clerk. Most of the important co-witnessing 'relationships' involve only a few key players, including Simon and Walter, as well as Herbert the dean, the clerk and canon Elias son of Fulbert, of Partick, the canon Bede, and the official Master John of Huntingdon. All of these men were active in either the reign of Bishop Herbert or Bishop Jocelin, or both.

Table 6.18. Most productive co-witnessing 'relationships' (H2/7)

Person 1	Person 2	#docs
Simon, archdeacon of Glasgow (fl.1165x74-1195x96)	Walter, clerk of Bishops Ingram and Jocelin	11
Elias of Partick, clerk, canon (son of Fulbert)	Simon, archdeacon of Glasgow (fl.1165x74-1195x96)	10
Elias of Partick, clerk, canon (son of Fulbert)	Herbert, dean of Glasgow (fl.1179x89-1204x07)	10
Bede, canon of Glasgow	Herbert, dean of Glasgow (fl.1179x89-1204x07)	10
Herbert, dean of Glasgow (fl.1179x89-1204x07)	John of Huntingdon, master, official of Glasgow (fl.1179x1208)	10
Elias of Partick, clerk, canon (son of Fulbert)	Walter, clerk of Bishops Ingram and Jocelin	9
Elias of Partick, clerk, canon (son of Fulbert)	Bede, canon of Glasgow	8
Simon, archdeacon of Glasgow (fl.1165x74-1195x96)	Herbert, dean of Glasgow (fl.1179x89-1204x07)	8
Walter, clerk of Bishops Ingram and Jocelin	Herbert, dean of Glasgow (fl.1179x89-1204x07)	8
Walter, clerk of Bishops Ingram and Jocelin	John of Huntingdon, master, official of Glasgow (fl.1179x1208)	8
Bede, canon of Glasgow	John of Huntingdon, master, official of Glasgow (fl.1179x1208)	8

Figure 6.43. Gephi: witnesses to documents of the bishops of Glasgow (H2/7)

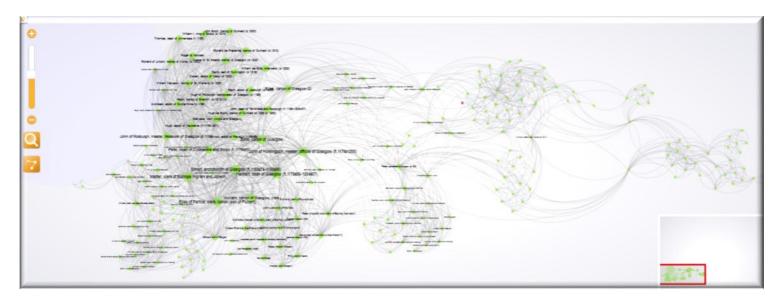


Figure 6.44. Gephi: witnesses to H7 documents, close-up

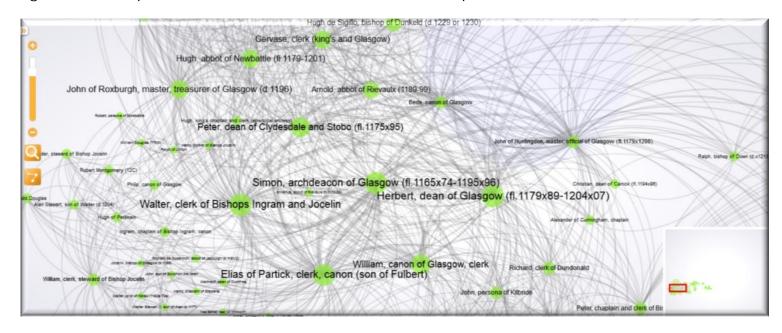
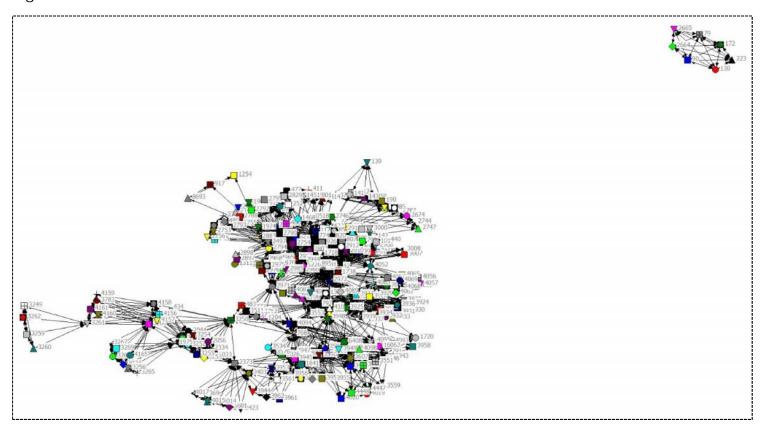


Figure 6.45. Netdraw: witnesses to H2/7 documents



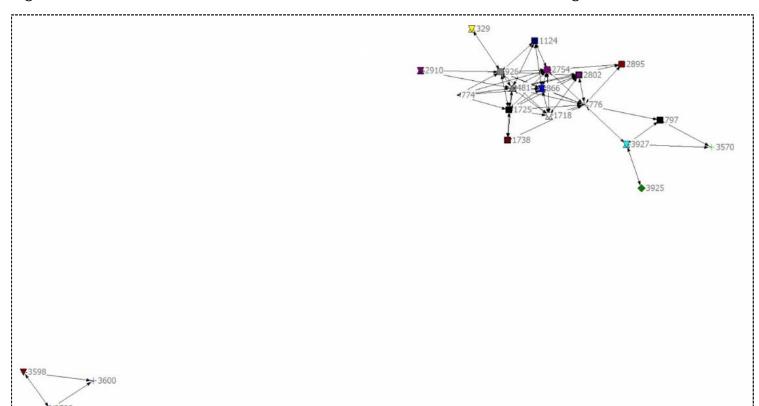


Figure 6.46. Netdraw: witnesses to H2/7 documents, >3 co-witnessing instances

The detached triad visible in the 'more than three' sociogram (Figure 6.46, above) constitutes the following individuals:

[3598] Ralph of Braid, master

[3599] Warin, clerk of Bishop Walter of Glasgow

[3600] Walter, clerk of Bishop Walter of Glasgow

These are men from the time of Bishop Walter of St Albans (1207-32).

Figure 6.47. Netdraw: witnesses to H2/7 documents, >5 co-witnessing instances

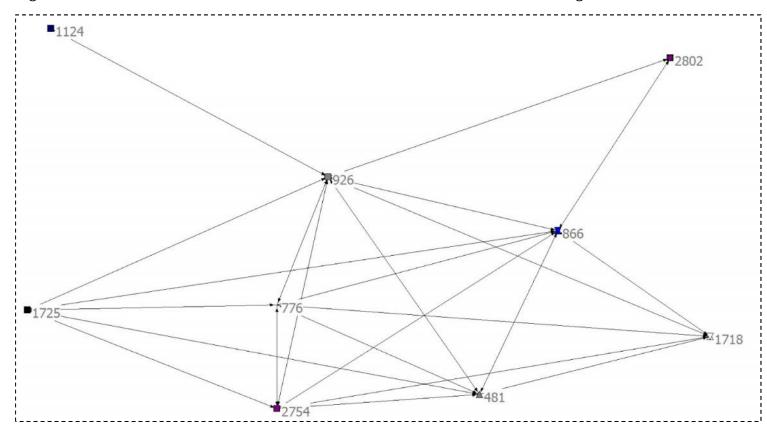
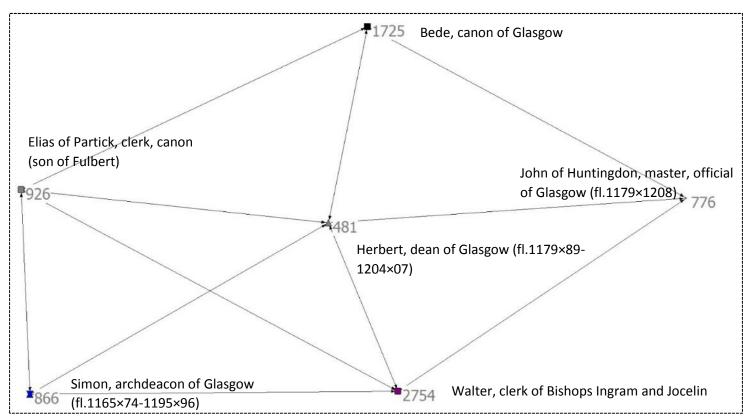


Figure 6.48. Netdraw: witnesses to H2/7 documents, >7 co-witnessing instances



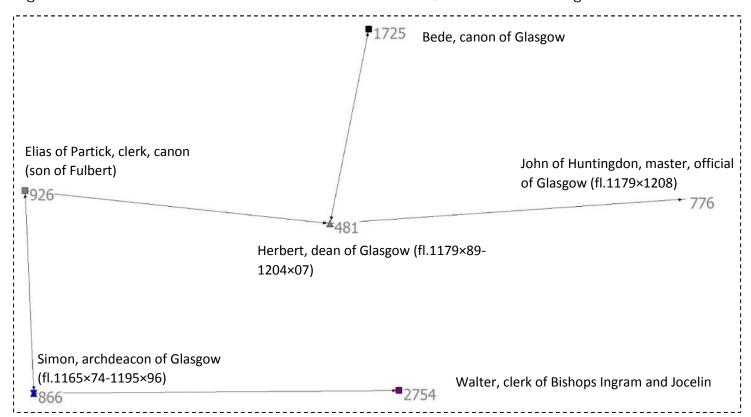


Figure 6.49. Netdraw: witnesses to H2/7 documents, >9 co-witnessing instances

3. Documents of the earls of Strathearn (H3/21)

There were 64 documents by the earls of Strathearn and their family members, defined by the H-number series 3/21. 62 of these are charters strictly speaking, one was a charter/brieve, and another a notification. There were 221 witnesses in the study, two of whom were women. There were 1765 edges in the SNA study. The period best represented among the most central individuals is the first half of the thirteenth century. Among laypeople, close relatives of the earls, as well as their stewards, have the highest centrality, while among clergy, the bishops and archdeacons of Dunblane, as well as churchmen associated with the comital caput of Crieff, are the most important. The individual with the highest centrality in terms of both degree and eigenvector was the steward Malise, son of Gilla na Naem, who witnessed charters for roughly four decades, from ca 1200 until the late 1230s. Malise's father, Gilla na Naem, and his son, Duncan, as well as the steward Brice of Dunning are also among the most central players. Other household members to appear are Henry son of Tristram, a rannaire, Joachim of Kinbuck, a knight, and William Hay, a clerk. The next highest person in terms of eigenvector comes in at only 88%, and that is Earl Gilbert of Strathearn's younger son, Malise. Other family members include Earl Gilbert's wife, Countess Matilda d'Aubigny, his sons Earl Robert, Gilbert, and

Fergus, and his brother Malise son of Earl Ferteth. Two bishops of Dunblane appear among the most central players – Abraham (1210×14 – 1223×25) and Clement (1233×58), as well as the archdeacon Gilbert. See also http://www.poms.ac.uk/social-network-analysis/private-charter-witnesses/earls-of-strathearn-h321/.

Table 6.19. Centrality: top 15 by degree

PoMS ID	Name	Degree	Eigenvector Centrality
3980	Malise, son of Gilla na Naem, steward of earls of Strathearn	80	1
749	Abraham, bishop of Dunblane (fl.1210×14-1220×25)	67	0.83937623
6889	Malise, son of Earl Gilbert of Strathearn (d.c.1272)	66	0.87981738
466	Gilbert, archdeacon of Dunblane (fl.1203×10-1235×39)	57	0.72994531
6976	Nicholas, son of Malise of Strathearn, chamberlain, rector of Crieff	57	0.35406347
2280	Gilbert or Gilla Brigte, son of Earl Gilbert, lord of Glencarnie (I) (d.a.1267)	55	0.71938914
3505	Robert, earl of Strathearn (1223-45)	50	0.72823605
4042	Gilla na Naem, steward of Earl Gilbert	50	0.65886962
3411	Brice, persona of Crieff	48	0.68970232
841	Malise, son of Ferteth earl of Strathearn (d.a.1214)	45	0.62328964
3497	Fergus, son of Gilbert, earl of Strathearn (d.c.1247)	45	0.72542091
4689	Duncan, son of Malise the steward	45	0.70925552
426	Matilda d'Aubigny, countess of Strathearn	44	0.61610913
1982	Clement, bishop of Dunblane (d.1258)	42	0.47514461
2360	Gilbert of Ruthven, lord of Ruthven in Strathearn	38	0.36600216
6870	Henry son of Tristram, rannaire	38	0.5844178
6974	Joachim of Kinbuck, knight	38	0.37305406
6950	Brice of Dunning, thane of Dunning, steward	37	0.45100816
6912	William Hay, clerk of earls of Strathearn	35	0.52871408

The most well-recorded co-witnessing 'relationships' involved churchmen. Of the 11 relationships which involved co-witnessing ten or more times, Bishop Abraham was a party to five of them, archdeacon Gilbert and Brice persona of Crieff were party to two each. These relationships attest to the exceptional production and survival of charters in the final two decades of Earl Gilbert's tenure, roughly the first two decades of the thirteenth century. Thus we have Earl Gilbert's brother, Malise, his son, Robert, and his steward, Malise, being represented so generously in this table.

Table 6.20. Most productive co-witnessing 'relationships' (H3/21)

Person 1	Person 2	#docs
Abraham, bishop of Dunblane (fl.1210×14-1220×25)	Malise, son of Gilla na Naem, steward of earls of Strathearn	17
Abraham, bishop of Dunblane (fl.1210×14-1220×25)	Brice, persona of Crieff	14
Abraham, bishop of Dunblane (fl.1210×14-1220×25)	Gilbert, archdeacon of Dunblane (fl.1203×10-1235×39)	12
Malise, son of Gilla na Naem, steward of earls of Strathearn	Brice, persona of Crieff	11
Malise, son of Gilla na Naem, steward of earls of Strathearn	Gilbert, archdeacon of Dunblane (fl.1203×10-1235×39)	11
Robert, earl of Strathearn (1223-45)	Abraham, bishop of Dunblane (fl.1210×14-1220×25)	11
Robert, earl of Strathearn (1223-45)	Malise, son of Gilla na Naem, steward of earls of Strathearn	11
Gilla na Naem, steward of Earl Gilbert	Malise, son of Ferteth earl of Strathearn (d.a.1214)	10
Malise, son of Ferteth earl of Strathearn (d.a.1214)	Constantine, judex (PER)	10
Abraham, bishop of Dunblane (fl.1210×14-1220×25)	Fergus, son of Gilbert, earl of Strathearn (d.c.1247)	10
Malise, son of Gilla na Naem, steward of earls of Strathearn	Malise, son of Earl Gilbert of Strathearn (d.c.1272)	10

Figure 6.50. Gephi: witnesses to documents of the earls of Strathearn (H3/21)

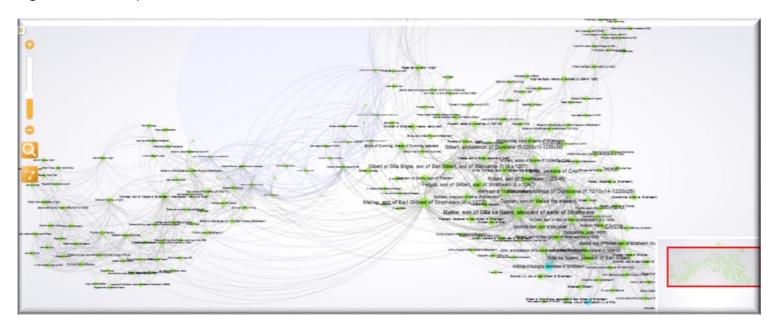


Figure 6.51. Gephi: witnesses to H3/21 documents, close-up

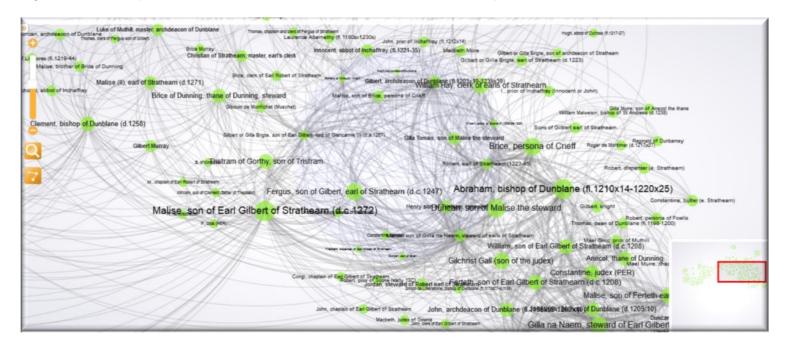
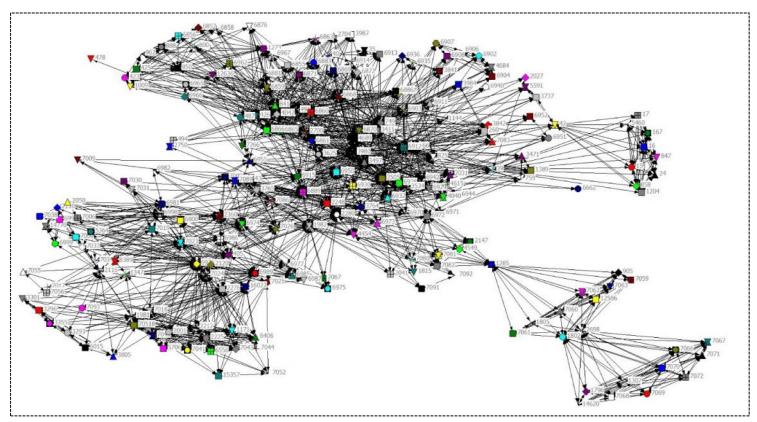


Figure 6.52. Netdraw: witnesses to H3/21 documents



The sociogram of more than 3 co-witnessing acts reveals the richness of the evidence for the early thirteenth century, whether in abundance or interconnectedness, compared to the later material. Earl Gilbert's son Malise [6889] is a vital connector to the later generations, represented by Bishop Clement [1982] and the knight Joachim of Kinbuck [6974]. The early thirteenth century period is explored in greater detail in the sociograms of individuals who co-witnessed more than seven and more than ten times, respectively.

Figure 6.53. Netdraw: witnesses to H3/21 documents, >3 co-witnessing instances

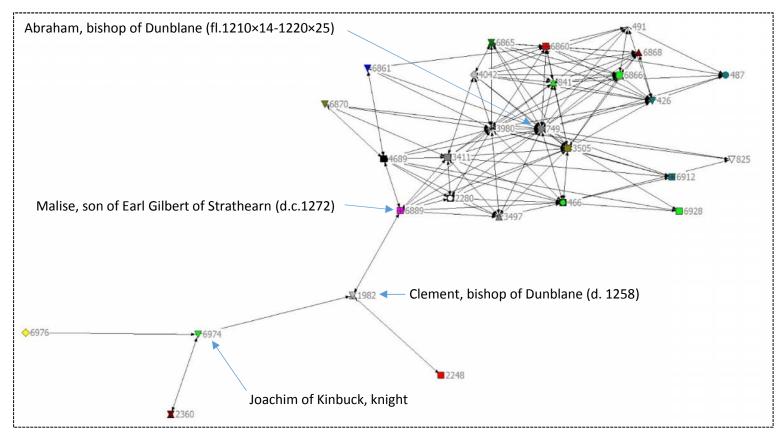


Figure 6.54. Netdraw: witnesses to H3/21 documents, >5 co-witnessing instances

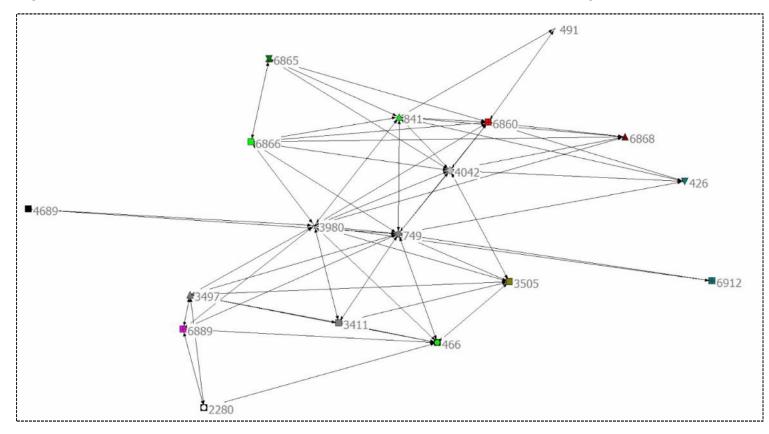


Figure 6.55. Netdraw: witnesses to H3/21 documents, >7 co-witnessing instances

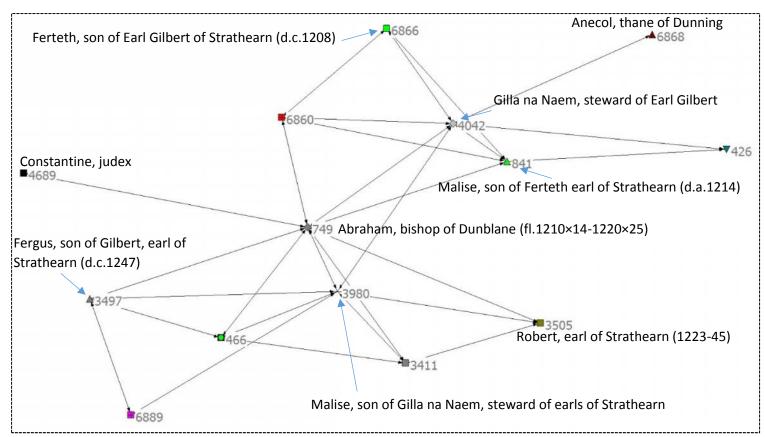
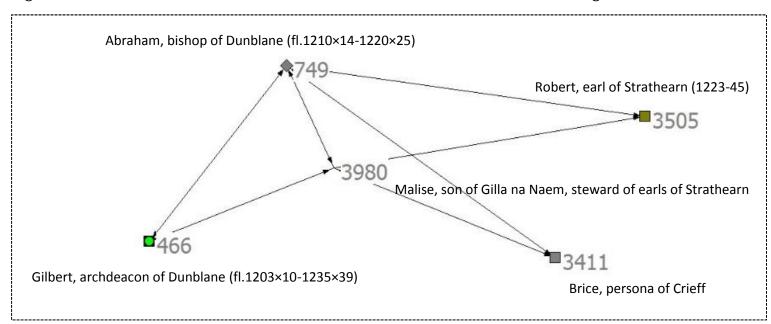


Figure 6.56. Netdraw: witnesses to H3/21 documents, >10 co-witnessing instances



4. Documents of the earls of Dunbar (H3/15)

There were 73 documents in the H3/15 series, representing the earls of Dunbar and their family. Of these one was a notification, the rest were charters. There were 362 witnesses –far more than in the Strathearn study. Three of these were women, all of whom were countesses. There were 3094 edges. See also http://www.poms.ac.uk/social-network-analysis/private-charter-witnesses/earls-of-dunbar-h315/

Figure 6.57. Gephi: witnesses to documents of the earls of Dunbar (H3/15).



Roger de Merlay (II) (d.c.1239)

Rajon de Aplindere

Miscour, cristophal (1900)

Rajon de Aplindere

Rajon de Rajon de Rajon

Rajon de

Figure 6.58. Gephi: witnesses to H3/15 documents, close-up

As with the Strathearn comital charters, the most prominent witnesses were family members and stewards. Indeed, stewards appear as even more important figures in the Dunbar charters. Roger de Merlay (no 1 – degree and eigenvector), Roland (no. 10 degree/ no. 7 eigenvector), and Stephen Papedy (no. 8 degree/ no. 10 eigenvector) were all stewards of Earl Patrick (I) at various points. The descendants of Aldan, who had been steward in the 1150s and 1160s, also remained very central to Dunbar earldom politics. Adam son of Aldan (no. 5 degree/ no. 8 eigenvector) and his son Patrick (no. 3 degree/ no. 4 eigenvector) seem to have maintained great importance despite apparently not holding onto the stewardship. The main family members to appear were Earl Patrick (I)'s sons Earl Patrick (II) [445] and William [4427]. Walter son of Edgar was also part of the comital kindred, as Edgar was Earl Patrick (I)'s great-uncle and a brother of the aforementioned Juliana. Other household members include a chaplain, Walter, and a clerk, Thomas Fraser. The family grouping which included the Frasers, Londons and Waughtons were prominent vassals, as was Ketill of Letham, whose name comes from the place now known as Leitholm near Coldstream.

Table 6.21. Centrality: top 15 by degree

PoMS ID	Name	Degree	Eigenvector
5781	Roger de Merlay (II) (d.c.1239) [steward]	110	1
445	Patrick (II), earl of Dunbar (d.1248)	93	0.915282
3149	Patrick, son of Adam son of Aldan the steward	92	0.906805
5798	Ness, son of Ness of Waughton	85	0.997628
4814	Adam, son of Aldan the Steward	78	0.668521
5787	Thomas Fraser, clerk of Earl Patrick	69	0.806917
4427	William, son of Earl Patrick (I) (d.1253)	67	0.477417
5912	Stephen Papedy, steward	67	0.571868
11520	Bernard Fraser (in ELO and BWK) (brother of Ness and John of London)	64	0.535782
5923	Roland, steward of Earl Patrick	60	0.698131
5925	Walter, chaplain (Dunbar)	58	0.524842
5789	Walter, son of Edgar	56	0.567614
5884	Ketill of Letham	54	0.478773
5901	Gilbert, son of Walter	54	0.548779
42	William del Bois, chancellor (d.1232)	53	0.766029

The most well-represented co-witnessing relationships involve some of the lay players already mentioned, including Bernard Fraser, Roger de Merlay (II), Patrick (II), earl of Dunbar, and his brother William. The only 'surprise' in the table below is the importance of Adam of Polwarth, son [2150], whose eigenvector was only about 40%, but who witnessed ten charters in the H3/15 series during the later part of the tenure of Earl Patrick (I). Polwarth is located in central Berwickshire and may have been held from the earl.

Table 6.22. Most productive co-witnessing 'relationships' (H3/15)

Person 1	Person 2	#docs
Bernard Fraser (in ELO and BWK) (brother of Ness and John of London)	Roger de Merlay (II) (d.c.1239)	9
Roger de Merlay (II) (d.c.1239)	Adam of Polwarth, son	9
Bernard Fraser (in ELO and BWK) (brother of Ness and John of London)	Adam of Polwarth, son	8
Patrick (II), earl of Dunbar (d.1248)	Roger de Merlay (II) (d.c.1239)	7
Patrick (II), earl of Dunbar (d.1248)	William, son of Earl Patrick (I) (d.1253)	7
Bernard Fraser (in ELO and BWK) (brother of Ness and John of London)	William, son of Earl Patrick (I) (d.1253)	6
Patrick (II), earl of Dunbar (d.1248)	Thomas Fraser, clerk of Earl Patrick	6
Roger de Merlay (II) (d.c.1239)	Patrick, son of Adam son of Aldan the steward	6
Roger de Merlay (II) (d.c.1239)	Thomas Fraser, clerk of Earl Patrick	6

Figure 6.59. Netdraw: witnesses to H3/15 documents

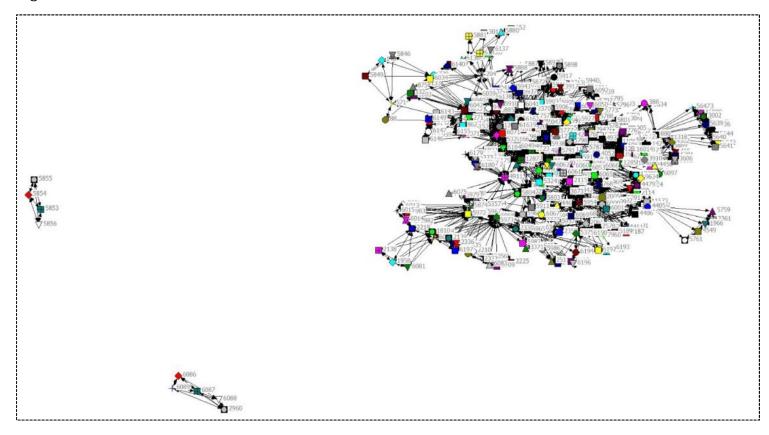


Figure 6.60. Netdraw: witnesses to H3/21 documents, >3 co-witnessing instances

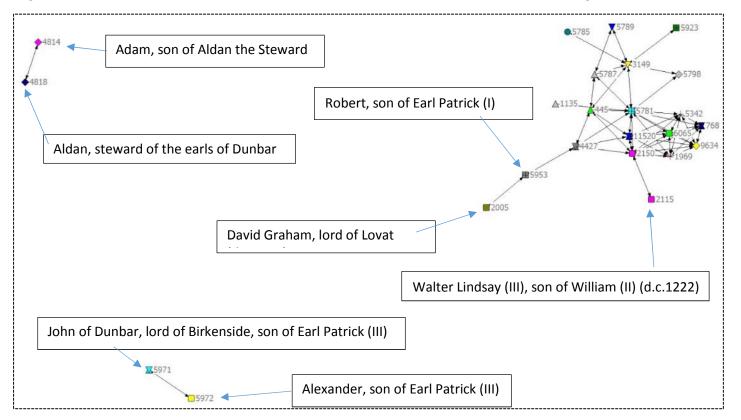


Figure 6.61. Netdraw: witnesses to H3/21 documents, >5 co-witnessing instances

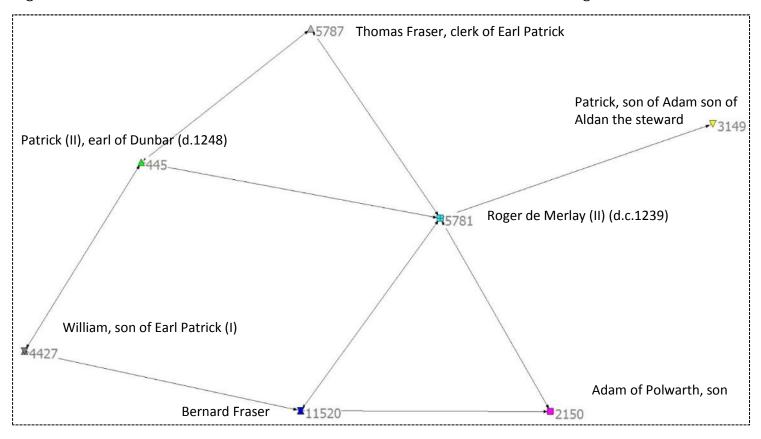
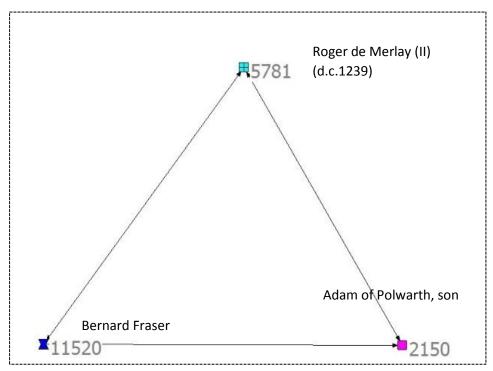


Figure 6.62. Netdraw: witnesses to H3/21 documents, >7 co-witnessing instances



PART FIVE: BENEFICIARY CASE STUDIES

1. Melrose Abbey

The studies of co-witnessing so far have all been defined by categories based on the grantors of documents, but it is also possible to defined case studies based on the beneficiary. Two examples are presented here: witnesses of documents for Melrose Abbey and for Arbroath Abbey, respectively. Melrose abbey was the beneficiary of 224 documents, all of which were charters strictly speaking (agreements with Melrose abbey have not been included here). There were 1017 witnesses connected by 10,570 edges in the study. The witnesses included three women. Of the 224 documents, 51 (22.8%) were royal, only 12 (5.4%) were by ecclesiastical grantors (H2), and 161 (71.9%) were by various sorts of lay grantors.

Jocelin, bishop of Glasgow (1175-99), in whose diocese Melrose lay, and who was himself a former abbot of Melrose, had by far the highest number of co-witnessing contacts, with a degree of 209. David, earl of Huntingdon (d. 1219) had the next highest degree (161) and the highest eigenvector score. Other Glasgow diocesan figures were very central in the Melrose abbey collection, including Simon, who was archdeacon during Jocelin's episcopate, and Master John of Huntingdon, who was the official. The episcopal clerks Walter [2754] and William [2790] also appeared in the top ten by degree. Among laymen, the sheriffs of Roxburgh John Maxwell [1281] and Bernard of Hadden [880] had relatively high degree centralities but unimpressive eigenvector scores of 35-40%. John Maxwell, however, had the highest betweenness centrality in the sociogram. While the most central figures were flourishing in the last quarter of the twelfth century, there is a fairly broad chronological range of individuals in the table of people in the top twenty by degree.

Table 6.23: Centrality: top 15 by degree

Poms	Person	Degree	Eigenvector	Betweenness
ID			Centrality	Centrality
745	Jocelin, bishop of Glasgow (d.1199)	209	0.969433071	55190.43371
142	David, earl of Huntingdon (d.1219)	161	1	26227.10592
866	Simon, archdeacon of Glasgow (fl.1165×74-1195×96)	140	0.848563015	11006.40339
776	John of Huntingdon, master, official of Glasgow (fl.1179×1208)	137	0.688349335	22666.81229
1281	John Maxwell, chamberlain, sheriff of Roxburgh (d.1241)	130	0.407692642	60477.63221
6060	Richard Nanus (le Nain)	114	0.46908129	26571.6271
15	Philip de Valognes, chamberlain (d.1215)	113	0.63269506	15362.94205

445	Patrick (II), earl of Dunbar (d.1248)	108	0.391641716	17738.12463
880	Bernard of Hadden, sheriff of Roxburgh	105	0.353307883	20098.24645
933	William of Hownam, son of John, son of Orm (d.1227)	102	0.476722667	15147.95263
42	William del Bois, chancellor (d.1232)	101	0.590488655	7613.795735
854	Gervase Avenel, lord of Eskdale (d.1219)	100	0.582574526	18122.57857
2754	Walter, clerk of Bishops Ingram and Jocelin	100	0.647206876	3278.949697
1285	Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	99	0.392555123	30344.93552
2790	William, clerk, steward of Bishop Jocelin	97	0.554695951	3657.78061
184	William de Somerville (I)	93	0.431304755	19958.59327
5781	Roger de Merlay (II) (d.c.1239)	93	0.318427303	27997.45148
3	Walter Stewart (I), son of Alan (d.1177)	91	0.496810807	13968.61104
797	Robert, archdeacon of Glasgow (d.1222)	90	0.499233889	9964.892652
13	Duncan (II), earl of Fife (d.1204)	89	0.522867701	4693.755745

The table of the most productive co-witnessing 'relationships' is an odd mélange, including Glasgow diocesan figures, members of the Ryedale family, earls of Dunbar, sheriffs of Roxburgh, and individuals from the founding era and first generation of the abbey's existence, like David I's constable, Hugh de Moreville, Bishop John of Glasgow, and William de Somerville. These represent various pockets of charter producing activity from different points in the abbey's history.

Table 6.24: Most productive co-witnessing 'relationships' (Melrose abbey)

Person1	Person2	Weight
Jocelin, bishop of Glasgow (d.1199	Simon, archdeacon of Glasgow (fl.1165x74-1195x96	11
Walter, son of Patrick of Ryedale	Ralph of Ryedale	11
Ralph of Ryedale	Robert of Barnoldby	11
Patrick (I), earl of Dunbar (d.1232	Patrick (II), earl of Dunbar (d.1248)	9
John Maxwell, chamberlain, sheriff of Roxburgh (d.1241)	Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242	9
John Maxwell, chamberlain, sheriff of Roxburgh (d.1241)	Bernard of Hadden, sheriff of Roxburgh	9
Walter, son of Patrick of Ryedale	Robert of Barnoldby	9
Hugh de Moreville (I) (d.1162)	William de Somerville (I)	8
Hugh de Moreville (I) (d.1162)	John, bishop of Glasgow (d.1147)	8
Hugh de Moreville (I) (d.1162)	Gervase Ridel, sheriff of Roxburgh (TRD	8
John, bishop of Glasgow (d.1147)	Gervase Ridel, sheriff of Roxburgh (TRD	8
Bernard of Hadden, sheriff of Roxburgh	William of Hownam, son of John, son of Orm (d.1227)	8
Richard Nanus (le Nain)	Ralph of Ryedale	8
Adam of Chatto	Ralph of Ryedale	8
Adam of Whitton	Ralph of Ryedale	8

Figure 6.63. Netdraw: witnesses to Melrose abbey beneficiary documents

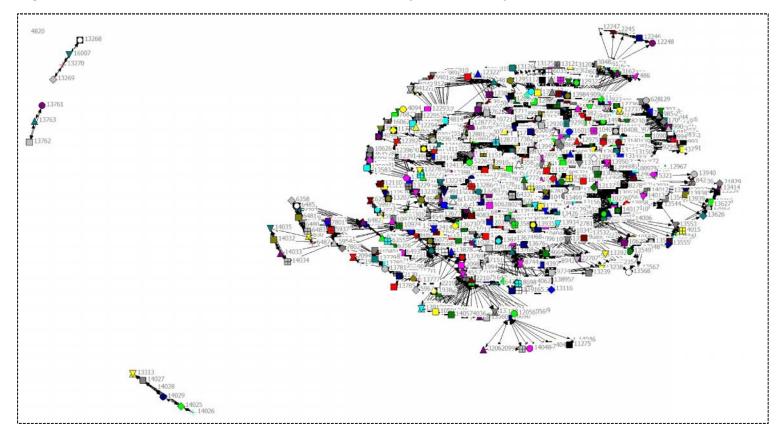


Figure 6.64. Netdraw: witnesses to Melrose abbey beneficiary documents, more than 3

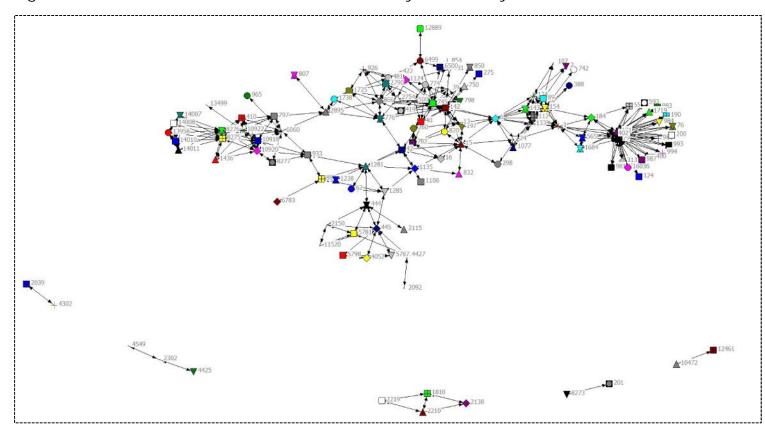


Figure 6.65. Netdraw: witnesses to Melrose abbey beneficiary documents, more than 5

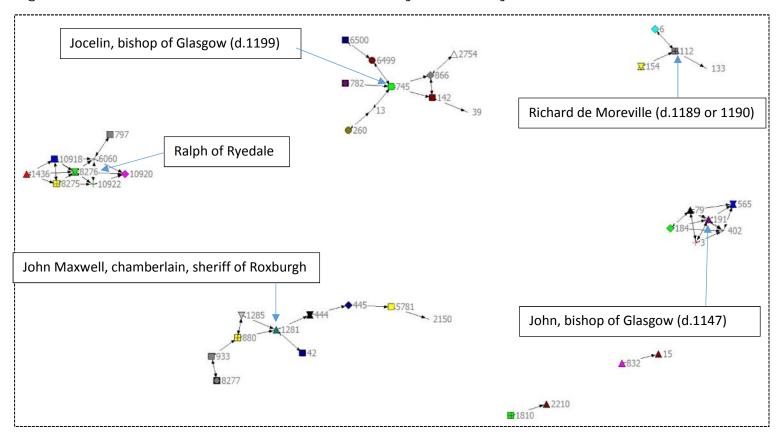
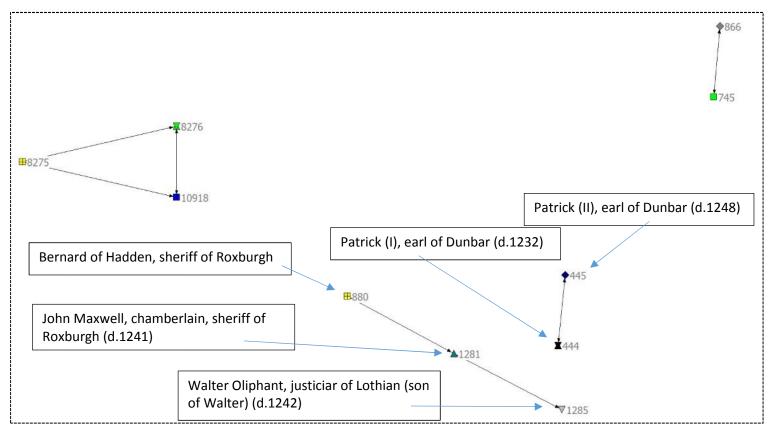


Figure 6.66. Netdraw: witnesses to Melrose abbey beneficiary documents, more than 8



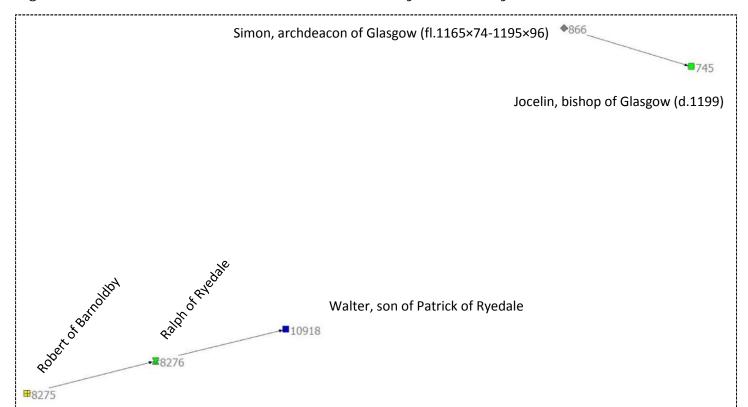


Figure 6.67. Netdraw: witnesses to Melrose abbey beneficiary documents, more than 10

2. Arbroath abbey

There are 229 documents in the study, all of which were charters. 77 (33.6%) of these are royal (H1) charters, 68 (30%) were ecclesiastical, mostly episcopal, charters (H2), and 84 (36.7%) of these are lay (H3) charters. The distribution of charters, then, are roughly equally distributed between royal, ecclesiastical, and lay charters, in stark contradistinction to the Melrose abbey case study. There 725 witnesses in these documents, one of whom was a woman. There were 6487 edges in the study.

William del Bois had the highest (by far) degree and betweeness centrality in the study, and also had the highest eigenvector. Most of the individuals with a degree over 100 were part of the royal inner circle in the part of William I's reign after 1178, when he founded the abbey. These include the chamberlain Philip de Valognes, the earl of Fife and royal justice Duncan (II), earl of Fife, William Hay, lord of Errol, and William Comyn, earl of Buchan: all of these names are familiar from our studies of royal charters. Hugh de Sigillo and Richard de Prebenda appear because of their periods as royal clerks.

The Arbroath abbey social network has been particularly influenced by the extremely enthusiastic charter production and preservation approach adopted by the abbey. For example, the abbey insisted on producing individual charters for each church and piece of land given, even when multiple gifts were

made by the same donor on the same occasion. For example, William Malveisin, bishop of St Andrews, issued twelve charters for Arbroath (H2/10/159-170), for which the following 14 individuals witnessed all of them, except for that Master Stephen of Lilliesleaf did not witness H2/10/160.

[42]	William del Bois, chancellor
[1215]	Simon, prior of St Andrews
[2491]	Stephen of Lilliesleaf, master, clerk, persona
[2971]	Peter, chaplain and clerk of Bishop Malveisin
[3511]	Michael, master, clerk, chaplain
[3838]	Adam Ovid, master
[3840]	Peter of Dryburgh, master, clerk
[3871]	Edward Murray, master, canon, bishop's clerk
[36]	Richard of Dover (Tyninghame), master, clerk
[48]	Simon de Noisy, clerk of Bishop William of St Andrews
[49]	William of Gullane, rector of Gullane
[759]	Denis, dean of Angus and Forfar
[835]	Laurence of Thornton, archdeacon of St Andrews

Similarly, a number of individuals appear multiple times as witnesses to six charters of Ralph, bishop of Brechin (H2/3/6-11). The following individuals witnessed all six of these charters:

[2590]	Andrew of Brechin, chaplain
[2614]	Peter, chaplain of bishops of Brechin
[2615]	Mael Brigte MacLeod, prior of céli De of Brechin
[2617]	Robert, son of Edgar
[2618]	Thomas, son of Robert son of Edgar
[42]	William del Bois, chancellor
[474]	Guy, abbot of Lindores
[59]	Gregory, bishop of Brechin

These two further individuals witnessed five of the six charters:

[39]	Hugh de Sigillo, bishop of Dunkeld
[2616]	Alexander Mowat, clerk, chaplain

Therefore, the importance of many individuals in this dataset has been amplified by the fact that in several cases, there are multiple documents recording a single event in time. It would be possible to create a bespoke case study which corrected for this trend by eliminating duplicate documents with identical witness lists, and this may be a methodology worth considering in the future.

Table 6.25. Centrality: top 20 by degree

PoMS ID	Person Name	Degree	Eigenvector Centrality	Betweenness Centrality
42	William del Bois, chancellor (d.1232)	206	1	51173.16
15	Philip de Valognes, chamberlain (d.1215)	114	0.875388	12310.79
24	William Hay (I), lord of Errol (d.c.1201)	114	0.971642	5205.113
13	Duncan (II), earl of Fife (d.1204)	107	0.879228	4106.727
39	Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	104	0.865837	5039.28
16	William Comyn, earl of Buchan (d.1233)	102	0.861498	5989.875
858	Walter of St Albans, bishop of Glasgow (d.1232)	101	0.791982	5982.088
798	Richard de Prebenda, bishop of Dunkeld (d.1210)	100	0.863934	2682.751
820	Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	94	0.838361	2024.025
1233	Philip Melville, justiciar of Scotia	92	0.395064	16987.13
474	Guy, abbot of Lindores (d.1219)	91	0.768695	5038.966
260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	90	0.802141	2184.466
59	Gregory, bishop of Brechin (fl.1189x98-1242x46)	89	0.438845	22558.97
782	Malcolm (I), earl of Fife (d.1229)	89	0.824851	3877.067
2	Matthew, bishop of Aberdeen (d.1199)	85	0.699893	4058.758
1106	Philip de Mowbray	83	0.635609	2304.971
419	Archibald, abbot of Dunfermline (d.1198)	82	0.706063	1387.851
66	David Hay, lord of Errol (d.1237�41)	79	0.740174	2810.178
4757	Adam, steward of Arbroath (son of Aldan)	78	0.347125	11881.29
307	Robert of London (d.1225)	74	0.640903	3692.103

The top co-witnessing relationships in the Arbroath abbey beneficiary study are all churchmen. The top five involve the longtime royal clerk and chancellor William del Bois [42]. William witnessed a number of ecclesiastical and lay charters in favour of Arbroath, in addition to royal charters. Most of the other churchmen to co-witness more than 13 times were personnel of the diocese of St Andrews, although the church of Brechin was also well represented, in the persons of Mael Brigte MacLeod, prior of the céli De of Brechin, and Gregory, the long-serving bishop of Brechin.

Table 6.26. Most productive co-witnessing 'relationships' (Arbroath abbey)

Person 1	Person 2	#docs
William del Bois, chancellor (d.1232)	Mael Brigte MacLeod, prior of celi De of Brechin	17
William del Bois, chancellor (d.1232)	Simon of St Andrews, master (fl.1199/1200-1212x18)	17
William del Bois, chancellor (d.1232)	Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	15
William del Bois, chancellor (d.1232)	Simon de Noisy, clerk of Bishop William of St Andrews	14
William del Bois, chancellor (d.1232)	William of Gullane, rector of Gullane	14
Simon de Noisy, clerk of Bishop William of St Andrews	William of Gullane, rector of Gullane	14
Gregory, bishop of Brechin (fl.1189x98-1242x46)	Alexander Mowat, clerk, chaplain (fl.1210s)	14
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Peter, chaplain and clerk of Bishop Malveisin	14
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Edward Murray, master, canon, bishop's clerk	14

Figure 6.68. Netdraw: witnessed to Arbroath abbey beneficiary documents

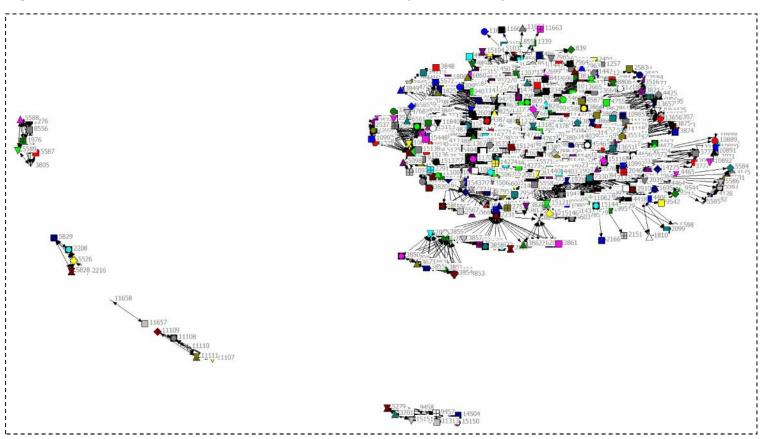


Figure 6.69. Netdraw: witnessed to Arbroath abbey beneficiary documents, more than 3

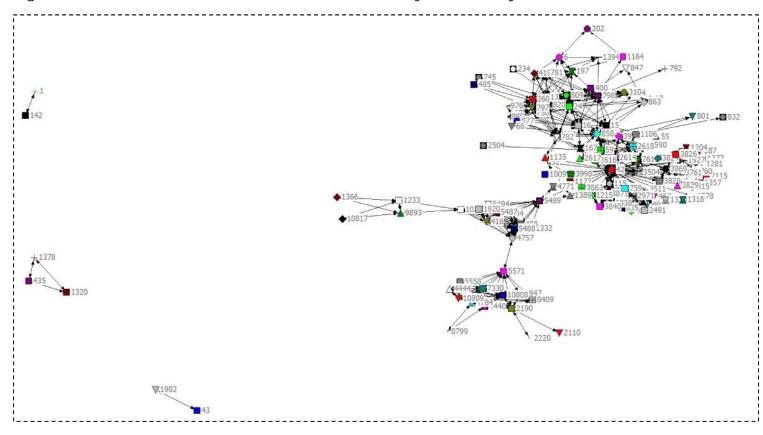


Figure 6.70. Netdraw: witnessed to Arbroath abbey beneficiary documents, more than 5

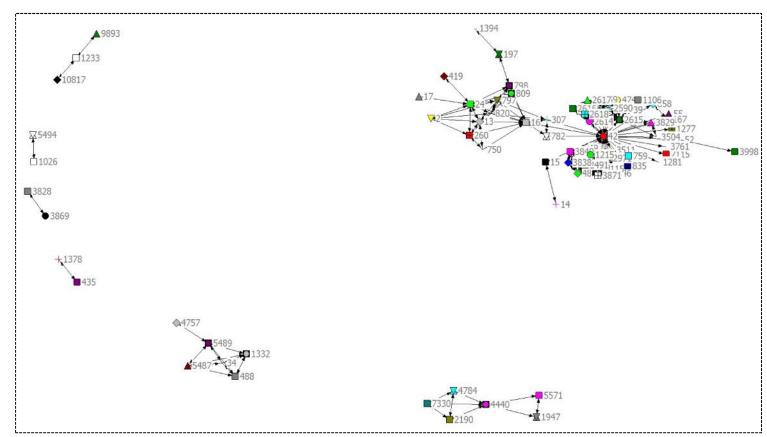


Figure 6.71. Netdraw: witnessed to Arbroath abbey beneficiary documents, more than 7

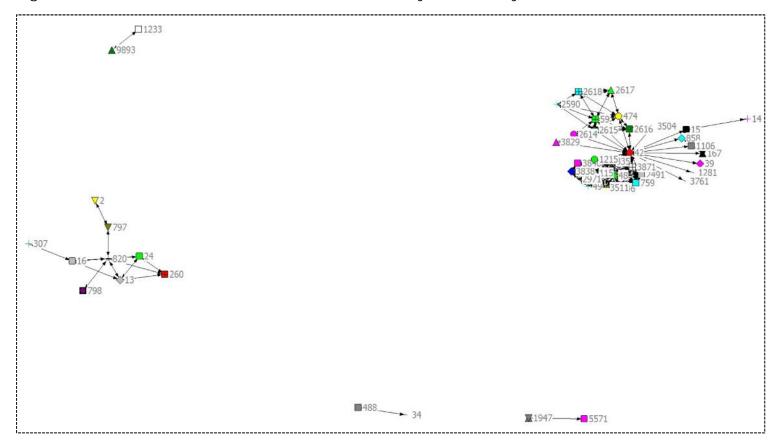


Figure 6.72. Netdraw: witnessed to Arbroath abbey beneficiary documents, more than 10

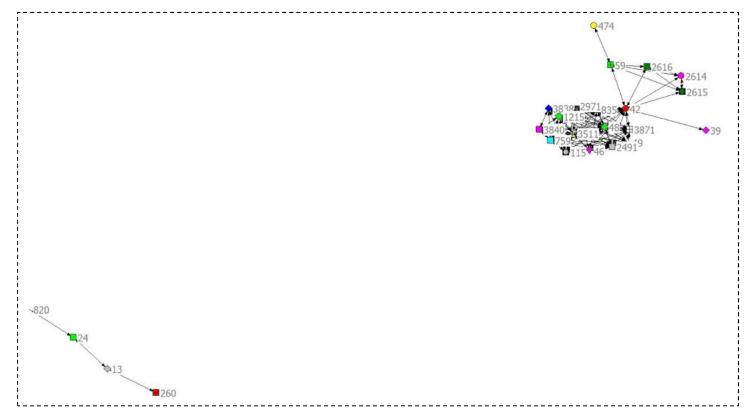


Figure 6.73. Netdraw: witnessed to Arbroath abbey beneficiary documents, more than 12

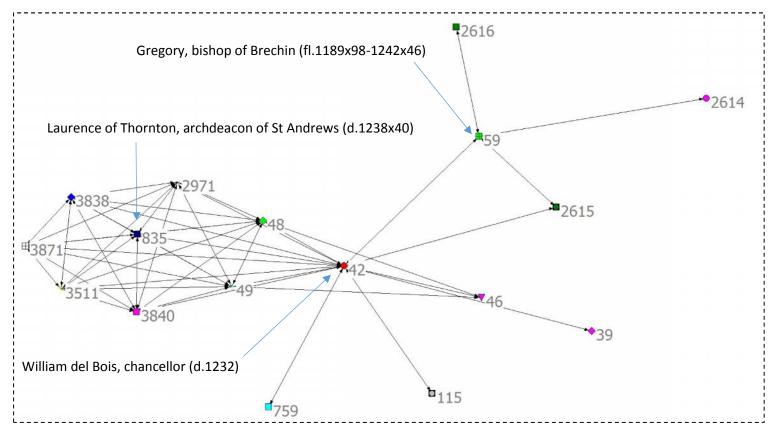
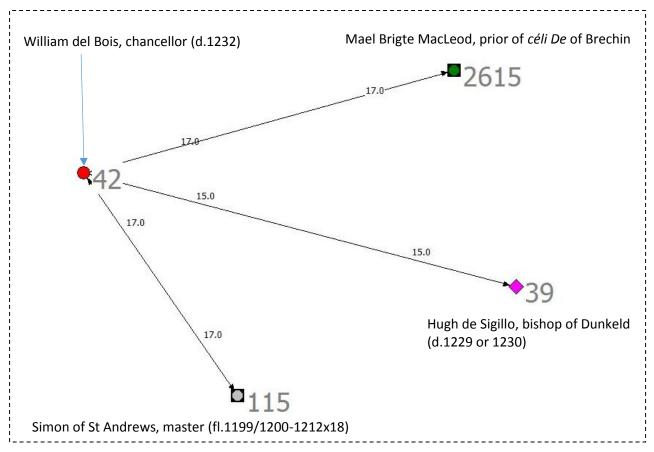


Figure 6.74. Netdraw: witnessed to Arbroath abbey beneficiary documents, more than 14



7 'SCOTLAND PROPER' DATASET

Introduction: What is the 'Scotland proper' dataset?

Up to the middle of the thirteenth century, the kings of Scots ruled over more a collection of divergent lands with different laws and customs than a single unified country. The most significant of these were generally seen as being Lothian, Strathclyde, and Galloway in the south of the kingdom, and 'Scotia' or 'Scotland proper' and Moray in the north. The term 'Scotland' was somewhat open-ended, but always referred to lands north of the Firth of Forth. As part of a broader historical analysis of a fairly tightly-defined 'Scotland proper' between the Rivers Forth and Spey (or more accurately, the counties of Banffshire, Aberdeenshire, Kincardineshire, Angus, Perthshire, Kinross-shire, Clackmannanshire, Fife, and the small part of Stirlingshire north of Forth), a bespoke dataset has been created out of the larger PoMS dataset which has been used thus far for our social network analyses. The documents in the 'Scotland proper' dataset have the same chronological limits as the main dataset, but only refer to possessions and privileges within 'Scotland proper', or have place-dates or other clear evidence that they were produced in 'Scotland proper'. This analysis of a major region within the kingdom thus offers us the potential for comparison with the broader PoMS dataset.

As Table 7.1 shows, there are 1841 documents of the five specified document types we have been looking at for all of our co-witnessing analyses. About 85% of these documents have witnesses, and have thus been included in the social network analysis. Of the 1572 documents with witnesses, as usual, the great majority (1473) are charters strictly speaking. About 89 percent of charters and charter/brieves had witnesses, about two-thirds of agreements had witnesses, but only about a third of settlements did. The proportions of various document types are shown in Figure 7.1. Table 7.2 breaks down the numbers according to grantor category. While 96% of royal documents had witnesses, only 76% of private charters did – this likely due to the witnesses not always being copied into cartularies. As Figure 7.2 shows 39 percent of the documents with witnesses were lay or private, 34% were royal, 22 percent had ecclesiastical grantors, and only 5 percent were two-sided documents.

Table 7.1. Breakdown of documents in 'Scotland proper' dataset

Document Type	Scotland proper (potential)	Scotland proper (docs with witnesses)	% docs with witnesses
Agreement	90	61	67.8%
Charter	1664	1473	88.5%
Charter/brieve	9	8	88.9%
Notification	36	16	44.4%
Settlement	42	14	33.3%
Totals	1841	1572	85.4%

Figure 7.1 Proportions of document types in 'Scotland proper' dataset

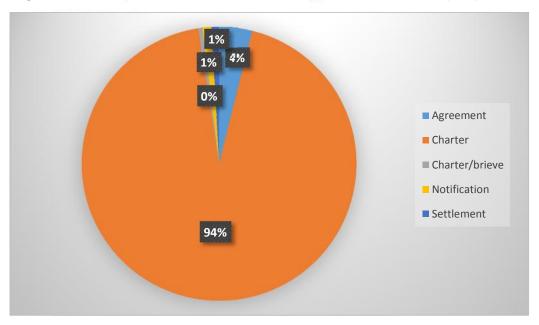


Table 7.2. Breakdown according to grantor category

Н	Grantor category	Scotland proper (potential)	Scotland proper (SNA dataset)	% docs with witnesses
H1/	Royal	555	534	96.2%
H2/	Ecclesiastical	454	347	76.4%
H3/	Lay or private	692	614	88.7%
H4/	Two-sided documents	140	77	55%
Totals		1841	1572	85.4%

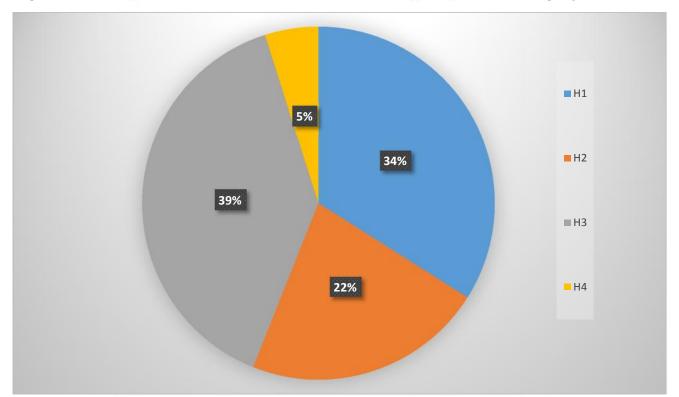


Figure 7.2. Proportions of SP documents according to grantor category

Table 7.3 lists the various kinds of primary transactions included in the dataset of Scotland proper documents, and the proportions of these are illustrated by Figure 7.3. The proportions are very similar to the dataset for the whole kingdom. Gifts and foundations amounted to 40.8%, as compared to 41.9% for the larger dataset, while confirmations were 21.4% and renewals were 15.9%, as compared to 19.6% and 13.6% for the whole kingdom, respectively. As in the larger study, the standard fodder for charters, that is to say, gifts, confirmations, renewals, quitclaims, successions and sales accounted for about 90% of all transactions.

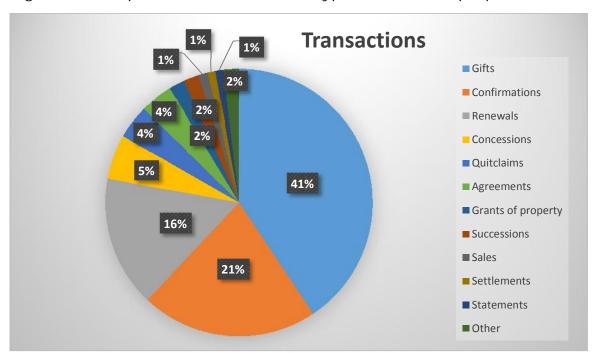


Figure 7.3. Proportions of transaction types in Scotland proper dataset

Table 7.3. Breakdown of transactions in study

Transaction type	Number of transactions	Percentage of transactions
Gifts and foundations ¹	641	40.8%
Confirmations	337	21.4%
Renewals	250	15.9%
Concessions ²	85	5.4%
Quitclaim & Resignation ³	65	4%
Agreements	62	3.9%
Grants of property (condedo)	30	1.9%
Successions	29	1.8%
Sales	18	1.1%
Settlements	15	
Statements ⁴	11	
Inspections	9	
Obligations	5	
Leases / wadset	5	
Institutions & ordination of vicarage	4	
Other/ misc.	7	
	1573	

¹ Plus one infeftment and three gifts (agreement)

² Including concession (agreements) the following follow same pattern

³ And renunciations of claim

⁴ Plus acknowledgement

There were 3834 witnesses, engaged in 13,590 acts of witnessing. This is slightly less than half of the numbers for the whole kingdom analysis, where there were 8967 witnesses in 31,448 acts of witnessing. Of these witnesses, 18 were women (see Table 7.3). There were 44,171 edges in the SNA sociogram.

Table 7.3. List of women who witnessed in the Scotland proper dataset

PoMS ID	Person
84	Ela, countess of Fife
95	Ada de Warenne (d.1178), countess of Northumberland
5497	Hextilda, countess of Atholl
1365	Margery, countess of Buchan (d.c.1244)
5508	Margaret, countess of Atholl
6663	Eleanor, daughter of William de Ferrers, wife of Roger de Quincy
426	Matilda d'Aubigny, countess of Strathearn
6855	Matilda, wife of Earl Malcolm (I) of Fife
1010	Agatha, wife of Humphrey Barclay
11464	Avice, daughter of Ela
1195	Eve, wife of William Hay, lord of Errol
11534	Ada, wife of Thomas Hay (12C)
6664	Orable, daughter of Ness son of William
13849	Margery Lindsay
14251	Margaret (mother of William de Valognes TRA3)
14254	Mary, sister of William de Valognes (TRA3)
6957	Soliva, wife of Robert of Meckphen
56	Ermengarde de Beaumont, queen of Scots (d.1233)

It is useful to compare the people with the highest betweenness centrality in the Scotland proper dataset with those for the whole country dataset (See Table 7.4, below). William del Bois, chancellor (d.1232), number one for the whole dataset, descends to the number 4 position in the SP dataset, while Duncan (II), earl of Fife (d.1204), number two in the whole dataset, rises to the number one spot here. Most of the other top individuals for the whole database are not among the top 20 in Scotland proper. This includes the stewards Alan and Walter (II), Earl Patrick (I) of Dunbar, the justiciar of Lothian Walter Oliphant (II), and bishops of Glasgow and Moray. The only other individuals from the top ten of the whole database in Table 7.4 are Malcolm (I), earl of Fife (d.1229) and John Hay (I), lord of Naughton (d.xOct.1266).

Table 7.4. Top 20 witnesses by betweenness (Scotland proper)

Rank	PoMS ID	Name	Betweenness
1	13	Duncan (II), earl of Fife (d.1204)	410757.9546
2	1389	John Hay (I), lord of Naughton (d.xOct.1266)	283253.5679
3	2	Matthew, bishop of Aberdeen (d.1199)	279272.0899
4	42	William del Bois, chancellor (d.1232)	263452.0193
5	5330	Henry, son of Geoffrey de Liberatione of Perth	191272.2987
6	3350	Adam of Makerstoun, master, provost (d.1280x86)	176565.4354
7	64	Henry of Stirling, son of Earl David	173445.9666
8	66	David Hay, lord of Errol (d.1237x41)	171602.6656
9	782	Malcolm (I), earl of Fife (d.1229)	167840.6777
10	260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	166924.7103
11	7030	Philip Oliphant, knight (13C)	165156.8340
12	2190	Robert Mowat, knight, justiciar, sheriff of Forfar	158562.3396
13	43	John, abbot of Lindores (fl.1219-44)	153754.6311
14	6889	Malise, son of Earl Gilbert of Strathearn (d.c.1272)	153125.0822
15	1971	Alan Durward (d.1275)	152253.9960
16	40	William Malveisin, bishop of St Andrews (d.1238)	147944.2254
17	5364	John Cameron, sheriff of Perth	143655.0134
18	1233	Philip Melville, justiciar of Scotia	143153.5671
19	11393	Alan, clerk (13C)	123196.3214
20	2067	Gilbert Hay (I), lord of Errol (d.1263) (son of David)	122034.1579

The individuals with the highest degree and eigenvector centralities in the Scotland proper dataset were active for the most part in the period between about 1170 and about 1230. Notable are a group of figures who were highly interconnected in the last quarter of the twelfth century, notably Duncan (II), earl of Fife (d.1204), Matthew, bishop of Aberdeen (d.1199), Gilbert or Gilla Brigte, earl of Strathearn (d.1223), Philip de Valognes, chamberlain (d.1215), William Hay (I), lord of Errol (d.c.1201). These men all had landholding and other interests in Fife, southern Perthshire, and Angus. They cowitnessed, especially in the context of the royal court, with contemporaries David, earl of Huntingdon (d.1219), Andrew, bishop of Caithness (d.1184), Ness, son of William, lord of Leuchars (d.1178x83) and Robert de Quincy (d.1200). The later part of King William's reign and the early part of Alexander II's reign were dominated by William del Bois, chancellor (d.1232), Malcolm (I), earl of Fife (d.1229), and especially William Comyn, earl of Buchan (d.1233). Most of the bishops listed in Table 7.5 had close royal connections, acting as royal clerks, chaplains, or chancellors.

Table 7.5. Top 21 witnesses by degree (Scotland proper)

Rank	PoMS ID	Name	Degree
1	13	Duncan (II), earl of Fife (d.1204)	487
2	2	Matthew, bishop of Aberdeen (d.1199)	425
3	260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	310
4	42	William del Bois, chancellor (d.1232)	309
5	782	Malcolm (I), earl of Fife (d.1229)	288
E	15	Philip de Valognes, chamberlain (d.1215)	255
7	16	William Comyn, earl of Buchan (d.1233)	250
8	66	David Hay, lord of Errol (d.1237x41)	241
8	24	William Hay (I), lord of Errol (d.c.1201)	241
10	798	Richard de Prebenda, bishop of Dunkeld (d.1210)	234
11	142	David, earl of Huntingdon (d.1219)	222
12	850	John Scott, bishop of Dunkeld (d.1203)	215
13	202	Andrew, bishop of Caithness (d.1184)	213
14	1389	John Hay (I), lord of Naughton (d.xOct.1266)	209
15	829	Ranulf de Wat, archdeacon of St Andrews (d.1209)	204
16	841	Malise, son of Ferteth earl of Strathearn (d.a.1214)	201
17	4	Ness, son of William, lord of Leuchars (d.1178x83)	200
18	14	Robert de Quincy (d.1200)	186
19	478	Henry, abbot of Arbroath (fl.1179-1207)	185
20	40	William Malveisin, bishop of St Andrews (d.1238)	178
20	39	Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	178

It should not be surprising by now that Duncan (II), earl of Fife (d.1204) occupies an unassailable spot in terms of the eigenvector centrality among Scotland proper documents. The exceptionally well-connected Matthew, bishop of Aberdeen, holds only an 87.5% score, with other key figures, such as William Comyn, earl of Buchan, and William del Bois, chancellor, managing only 61.6% and 56.7%, respectively. There are no great surprises on the eigenvector league table for Scotland proper as it is largely occupied by the same familiar names as the degree centrality table. Most of these men were also fairly prominent in the SNA study of the whole database.

Table 7.6. Top 20 witnesses by eigenvector (Scotland proper)

Rank	PoMS ID	Name	Eigenvector
1	13	Duncan (II), earl of Fife (d.1204)	1
2	2	Matthew, bishop of Aberdeen (d.1199)	0.875450565
3	260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	0.760963387
4	782	Malcolm (I), earl of Fife (d.1229)	0.685512636
5	24	William Hay (I), lord of Errol (d.c.1201)	0.668670027
6	15	Philip de Valognes, chamberlain (d.1215)	0.661281159
7	798	Richard de Prebenda, bishop of Dunkeld (d.1210)	0.655126297
8	16	William Comyn, earl of Buchan (d.1233)	0.61644977
9	142	David, earl of Huntingdon (d.1219)	0.604677418
10	850	John Scott, bishop of Dunkeld (d.1203)	0.57034643
11	42	William del Bois, chancellor (d.1232)	0.56772836
12	66	David Hay, lord of Errol (d.1237x41)	0.543237506
13	14	Robert de Quincy (d.1200)	0.53934394
14	202	Andrew, bishop of Caithness (d.1184)	0.524489101
15	1	William I, king of Scots (d.1214)	0.521118205
16	110	Gilla Brigte, earl of Angus (d.x1189)	0.517727468
17	478	Henry, abbot of Arbroath (fl.1179-1207)	0.501802042
18	39	Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	0.493999729
19	820	Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	0.490623259
20	185	Geoffrey (I) Melville	0.490281247

The Netdraw sociogram of all witnesses to the Scotland proper SNA study is familiar as a slightly thinned-out version of the 'fish' sociogram (see Figure 4.4). At the level of more than 10 witnessing acts (Figure 7.6), it is possible to make out a main segment of nodes bearing some resemblance to the three-pronged structure in the larger study (see Figures 4.7 and 4.8). While the chronological sweep is still evident, with earlier people on the right, a large agglomeration of nodes during the reign of William I, and the central positions of [13] Earl Duncan and [42] William del Bois evident, what is immediately noticeable is the disappearance of the large segment of Coldingham witnesses branching off towards the bottom of the page. The close-up of this segment in the Scotland proper at the level of witnessing more than 15 times (Figure 7.8) is most comparable to the study of more than 20 witnessing acts in the larger dataset.

Figure 7.4. Netdraw: 'Scotland proper' dataset witnesses

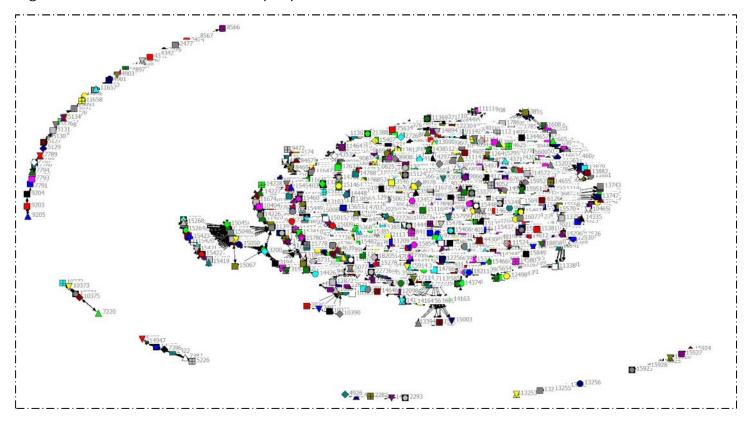


Figure 7.5. Netdraw: 'Scotland proper', more than 5 co-witnessing acts

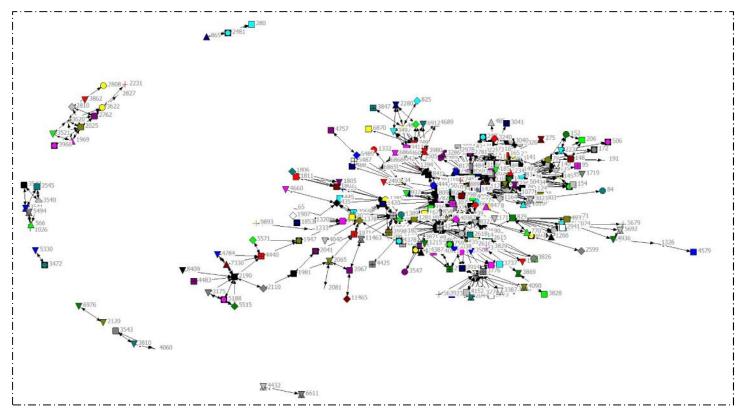


Figure 7.6. Netdraw: 'Scotland proper', more than 10 co-witnessing acts

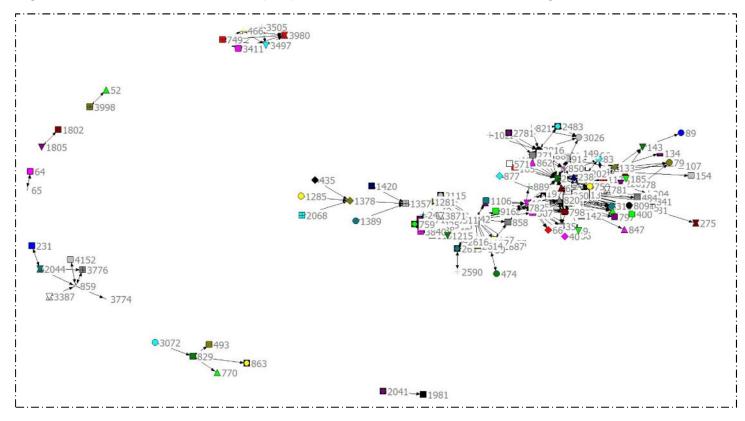
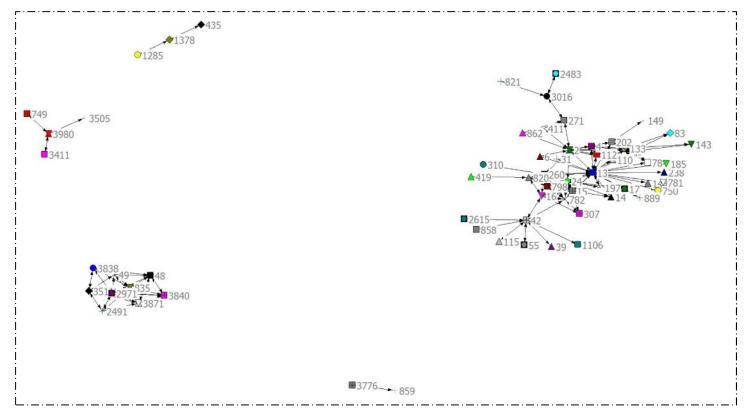


Figure 7.7. Netdraw: 'Scotland proper', more than 15 co-witnessing acts



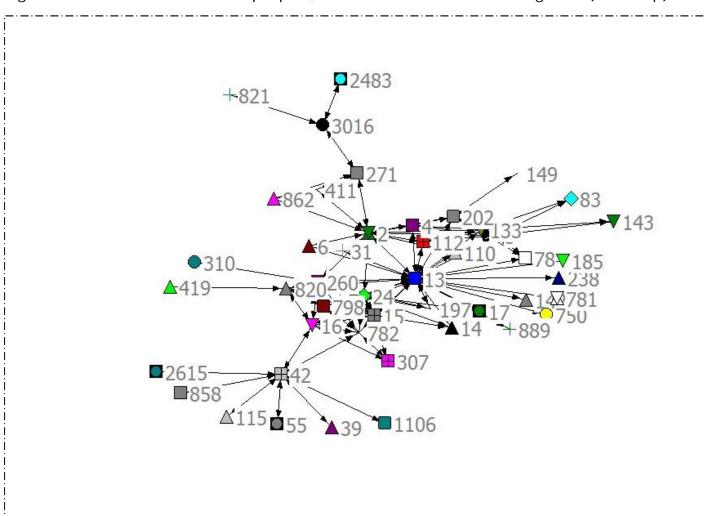
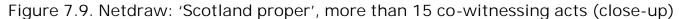
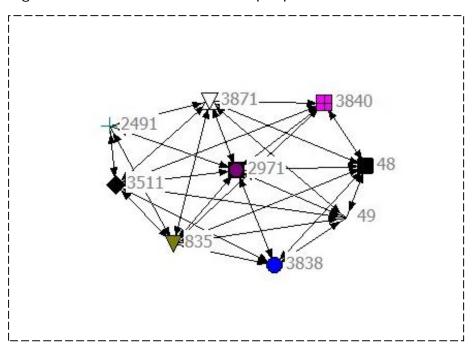


Figure 7.8. Netdraw: 'Scotland proper', more than 15 co-witnessing acts (close-up)





All of the individuals who co-witnessed more than fifteen times are listed in Table 7.7 along with their centrality numbers at that level. Figure 7.9 offers a close-up of the second-largest segment of the study of individuals who co-witnessed more than 15 times. These men were all part of the St Andrews church establishment during the episcopate of Bishop William Malveisin (1202-38). The segment represents a highly interconnected group of triads and cliques. Of the 9 men, all of the other eight are linked to Peter the chaplain, and most of the men have witnessed more than fifteen times with most of the others in the segment.

[48]	Simon de Noisy, clerk of Bishop William of St Andrews
[49]	William of Gullane, rector of Gullane
[835]	Laurence of Thornton, archdeacon of St Andrews (d.1238x40)
[2491]	Stephen of Lilliesleaf, master, clerk, persona
[2971]	Peter, chaplain and clerk of Bishop Malveisin
[3511]	Michael, master, clerk, chaplain (fl.1201-1220x25)
[3838]	Adam Ovid, master (fl.1203-33)
[3840]	Peter of Dryburgh, master, clerk
[3871]	Edward Murray, master, canon, bishop's clerk

The largest segment of people who co-witnessed more than 15 times. The position of [13] Duncan (II), earl of Fife, is very significant. In betweenness at this level, his score of 564 is more than twice that of [2] Bishop Matthew of Aberdeen (261), [42] William del Bois (249), and [16] William Comyn earl of Buchan (146). Earl Duncan co-witnessed with 26 people more than 15 times in the Scotland proper database, while Bishop Matthew only co-witnessed with 13, and William Comyn and William del Bois only witnessed alongside eight others. The high eigenvector and degree scores of Duncan, Matthew, [24] William Hay and [260] Earl Gilbert – the only four to be connected to at least ten other witnesses, and to have eigenvector scores of over 50% - underline the importance of the network which bound together these four power-players in the central area of Southern Perthshire and Fife. This is demonstrated in Figure 7.16, which shows that William Hay and Earl Duncan witnessed together 53 times, Bishop Matthew and Earl Duncan witnessed together 57 times, and Earl Gilbert and Earl Duncan witnessed together 61 times.

Table 7.7. Centrality: network of people who co-witness more than 15 times

PoMS ID	Name	Degree	Betweeness	Eigenvector
13	Duncan (II), earl of Fife (d.1204)	26	563.8167	1
2	Matthew, bishop of Aberdeen (d.1199)	13	260.75	0.605126
24	William Hay (I), lord of Errol (d.c.1201)	12	80.233	0.642807
260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	10	82.1	0.580194
3	Walter Stewart (I), son of Alan (d.1177)	9	47.75	0.437621
16	William Comyn, earl of Buchan (d.1233)	8	145.5167	0.472665
42	William del Bois, chancellor (d.1232)	8	249	0.13706
133	Nicholas of Roxburgh, chancellor (d.1171?)	8	42.25	0.412351
835	Laurence of Thornton, archdeacon of St Andrews (d.1238×40)	8	1.9833	0.191163
2971	Peter, chaplain and clerk of Bishop Malveisin	8	1.9833	0.191163
202	Andrew, bishop of Caithness (d.1184)	7	47	0.385039
820	Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	7	44.45	0.434888
782	Malcolm (I), earl of Fife (d.1229)	7	128.6667	0.421234
3511	Michael, master, clerk, chaplain (fl.1201-1220×25)	7	1.4833	0.170497
3871	Edward Murray, master, canon, bishop's clerk	7	1.2	0.172998
15	Philip de Valognes, chamberlain (d.1215)	6	3.0667	0.415068
48	Simon de Noisy, clerk of Bishop William of St Andrews	6	0.65	0.152755
49	William of Gullane, rector of Gullane	6	0.3667	0.157347
4	Ness, son of William, lord of Leuchars (d.1178×83)	5	0	0.353743
798	Richard de Prebenda, bishop of Dunkeld (d.1210)	5	0.4	0.377853
3838	Adam Ovid, master (fl.1203-33)	5	0.166667	0.13398
3840	Peter of Dryburgh, master, clerk	5	0.166667	0.136408
112	Richard de Moreville (d.1189 or 1190)	4	0	0.305625
271	Robert, son of Saewulf, bishop's chancellor	4	126	0.110872
2491	Stephen of Lilliesleaf, master, clerk, persona	4	0	0.112624
14	Robert de Quincy (d.1200)	3	0	0.252535
78	Walter de Bidun (d.1178)	3	0	0.226673
411	Andrew, archdeacon of Lothian (fl.1147×59-1178×84)	3	0	0.106591
3016	Alexander, chaplain of bishops of St Andrews (12C)	3	87	0.021168
862	Herbert Scott, master, clerk (fl.1144×59-1172×78)	3	0	0.106591
197	William Lindsay (II) (d.c.1205)	3	0	0.272732
307	Robert of London (d.1225)	3	0	0.188584
3980	Malise, son of Gilla na Naem, steward of earls of Strathearn	3	3	0.00708
17	John (I) Hastings, sheriff (12/13C)	2	0	0.202074
110	Gilla Brigte, earl of Angus (d.×1189)	2	0	0.199233
143	Ingram, bishop of Glasgow (d.1174)	2	0	0.106392
83	David Oliphant (12C)	2	0	0.106392
6	Walter Barclay, chamberlain (d.c.1193)	2	0	0.199233
1378	Walter Stewart (II), son of Alan (d.1241)	2	1	0.004363
39	Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	1	0	0.020736
55	Richard Revel, lord of Coultra (d.1215×25)	1	0	0.020736
142	David, earl of Huntingdon (d.1219)	1	0	0.123755

149	Gregory, bishop of Dunkeld (d.1169)	1	0	0.048118
238	Malcolm, earl of Atholl (d.c.1197)	1	0	0.123755
185	Geoffrey (I) Melville	1	0	0.123755
31	Hugh Gifford, lord of Yester	1	0	0.123755
419	Archibald, abbot of Dunfermline (d.1198)	1	0	0.053058
310	William de Moreville (d.1196)	1	0	0.070658
750	Robert Barclay, brother of Walter	1	0	0.123755
781	Richard of Lincoln, bishop of Moray (d.1203)	1	0	0.123755
889	Adam of Ceres, knight (fl.1154×1200)	1	0	0.123755
858	Walter of St Albans, bishop of Glasgow (d.1232)	1	0	0.020736
1106	Philip de Mowbray	1	0	0.020736
1285	Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	1	0	0.002829
821	Walter of Roxburgh, archdeacon of St Andrews (fl.1165×72-1179×88)	1	0	0.005371
2615	Mael Brigte MacLeod, prior of <i>céli De</i> of Brechin	1	0	0.020736
749	Abraham, bishop of Dunblane (fl.1210×14-1220×25)	1	0	0.003702
2483	Gamel, doorward, master (St Andrews)	1	0	0.005371
3411	Brice, persona of Crieff	1	0	0.003702
859	Alexander de St Martin, master (fl.1214×40-1247)	1	0	0.002008
3776	Hugh of Melburne, master	1	0	0.002008
115	Simon of St Andrews, master (fl.1199/1200-1212×18)	1	0	0.020736
3505	Robert, earl of Strathearn (1223-45)	1	0	0.003702
435	William of Bondington, bishop of Glasgow (d.1258)	1	0	0.002829

Figure 7.10. Netdraw: 'Scotland proper', more than 20 co-witnessing acts

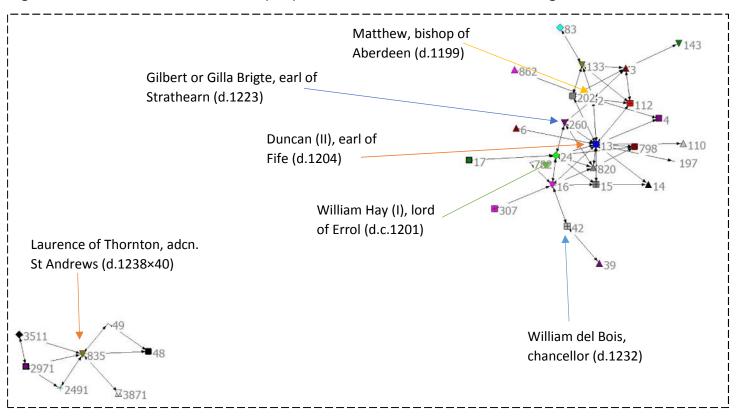


Figure 7.11. Netdraw: 'Scotland proper', more than 25 co-witnessing acts

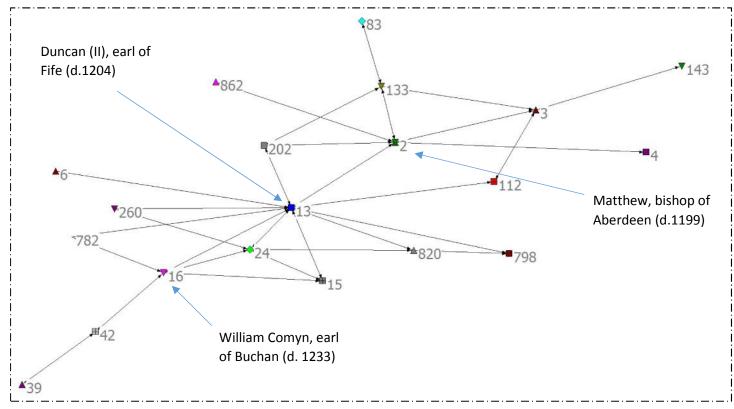


Figure 7.12. Netdraw: 'Scotland proper', more than 30 co-witnessing acts

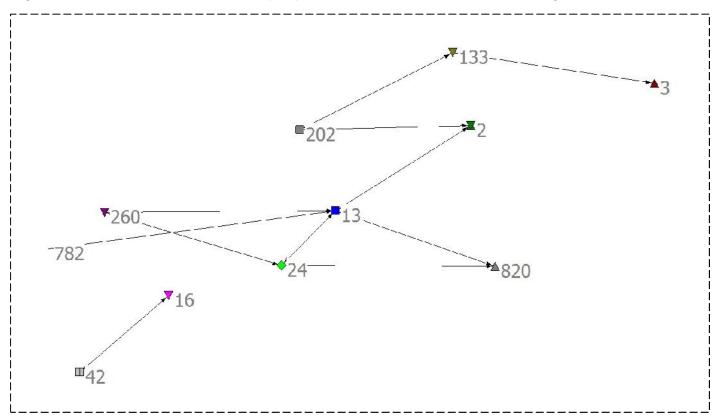


Figure 7.13. Netdraw: 'Scotland proper', more than 35 co-witnessing acts

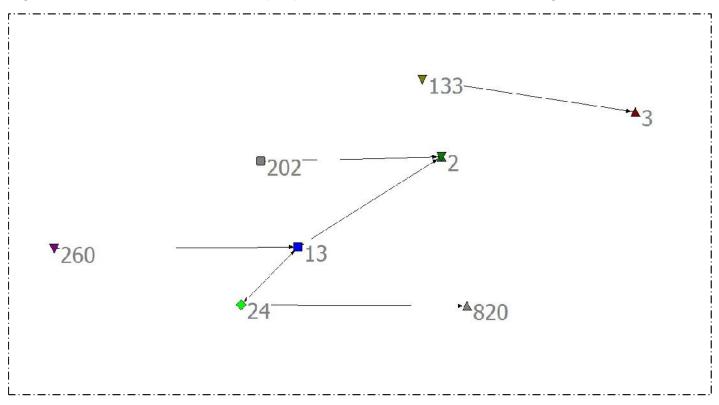


Figure 7.14. Netdraw: 'Scotland proper', more than 35 co-witnessing acts

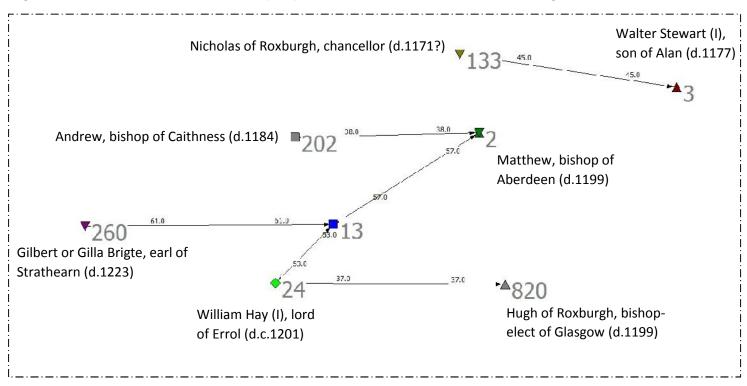


Figure 7.15. Netdraw: 'Scotland proper', more than 45 co-witnessing acts

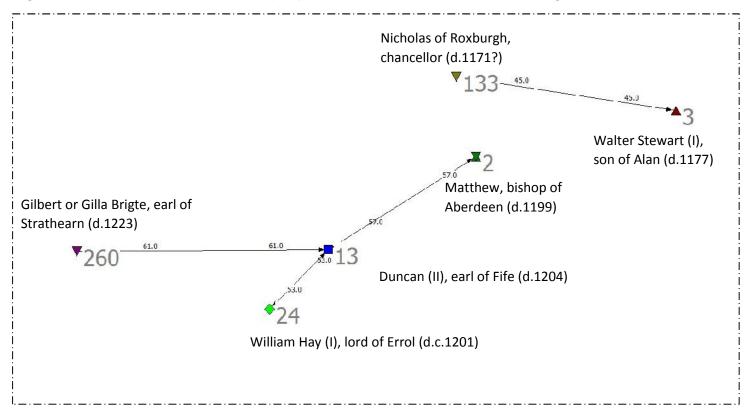


Figure 7.16. Netdraw: 'Scotland proper', more than 50 co-witnessing acts

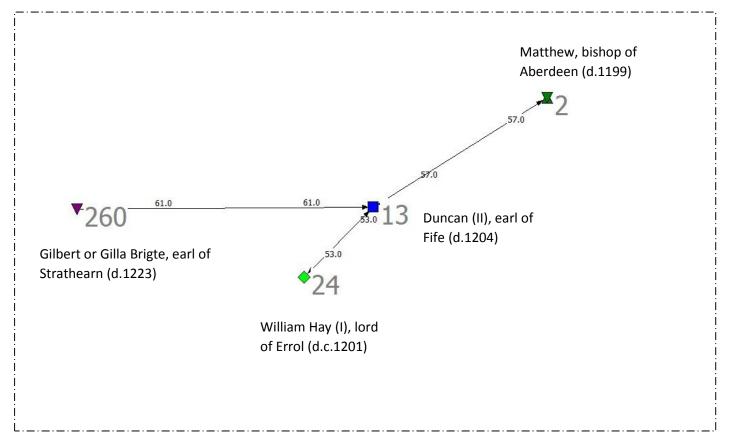
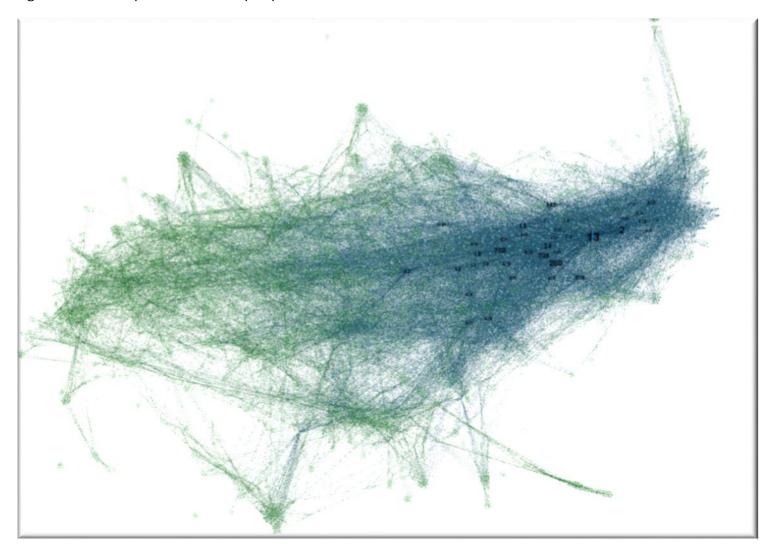


Table 7.8. Most productive co-witnessing 'relationships' (25 and above)

Person 1	Person 2	Docs witnessed
Duncan (II), earl of Fife (d.1204)	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	61
Duncan (II), earl of Fife (d.1204)	Matthew, bishop of Aberdeen (d.1199)	57
Duncan (II), earl of Fife (d.1204)	William Hay (I), lord of Errol (d.c.1201)	53
Walter Stewart (I), son of Alan (d.1177)	Nicholas of Roxburgh, chancellor (d.1171?)	45
Matthew, bishop of Aberdeen (d.1199)	Andrew, bishop of Caithness (d.1184)	38
William Hay (I), lord of Errol (d.c.1201)	Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	37
William Hay (I), lord of Errol (d.c.1201)	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	33
William Comyn, earl of Buchan (d.1233)	William del Bois, chancellor (d.1232)	32
Nicholas of Roxburgh, chancellor (d.1171?)	Andrew, bishop of Caithness (d.1184)	32
Duncan (II), earl of Fife (d.1204)	Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	31
Duncan (II), earl of Fife (d.1204)	Malcolm (I), earl of Fife (d.1229)	31
Duncan (II), earl of Fife (d.1204)	William Comyn, earl of Buchan (d.1233)	30
Duncan (II), earl of Fife (d.1204)	Andrew, bishop of Caithness (d.1184)	30
Duncan (II), earl of Fife (d.1204)	Walter Barclay, chamberlain (d.c.1193)	30
William Comyn, earl of Buchan (d.1233)	William Hay (I), lord of Errol (d.c.1201)	30
Philip de Valognes, chamberlain (d.1215)	William Hay (I), lord of Errol (d.c.1201)	29
Matthew, bishop of Aberdeen (d.1199)	Ness, son of William, lord of Leuchars (d.1178x83)	29
Duncan (II), earl of Fife (d.1204)	Philip de Valognes, chamberlain (d.1215)	28
Philip de Valognes, chamberlain (d.1215)	William Comyn, earl of Buchan (d.1233)	28
Richard de Moreville (d.1189 or 1190)	Walter Stewart (I), son of Alan (d.1177)	28
Matthew, bishop of Aberdeen (d.1199)	Nicholas of Roxburgh, chancellor (d.1171?)	28
Matthew, bishop of Aberdeen (d.1199)	Herbert Scott, master, clerk (fl.1144x59-1172x78)	28
Nicholas of Roxburgh, chancellor (d.1171?)	David Oliphant (12C)	28
Duncan (II), earl of Fife (d.1204)	Richard de Moreville (d.1189 or 1190)	26
Duncan (II), earl of Fife (d.1204)	Richard de Prebenda, bishop of Dunkeld (d.1210)	26
William Comyn, earl of Buchan (d.1233)	Malcolm (I), earl of Fife (d.1229)	26
Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	William del Bois, chancellor (d.1232)	26
Matthew, bishop of Aberdeen (d.1199)	Walter Stewart (I), son of Alan (d.1177)	26
Walter Stewart (I), son of Alan (d.1177)	Ingram, bishop of Glasgow (d.1174)	26
Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	Richard de Prebenda, bishop of Dunkeld (d.1210)	26
Duncan (II), earl of Fife (d.1204)	Ness, son of William, lord of Leuchars (d.1178x83)	25
Matthew, bishop of Aberdeen (d.1199)	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	25

Duncan (II) earl of Fife was part of twelve pairs (37.5%) in the above table, and eight of the 15 pairs with 30 or more co-witnessing acts. Bishop Matthew was part of seven pairs. By contracts, William Comyn was part of five pairs and Philip de Valognes was part of only three pairs. The earlier generation is also represented here, with Walter son of Alan (I) (d. 1177) part of four pairs and Andrew, bishop of Caithness (d. 1184) part of three.

Figure 7.17. Gephi: Scotland proper, all witnesses



The following series of Gephi sociograms allows us to visualize the most central players in the Scotland proper dataset against the backdrop of all the nodes in the network, with the chronological sweep going from mid-twelfth century on the right to late thirteenth century on the left. The label size reflects the eigenvector centrality of the witnesses. This approach is particularly valuable at the level of thirty or more co-witnessing acts, because it illustrates the positions of top players chronologically, from [3] Walter Stewart (I) (d. 1177) through to [42] William del Bois, chancellor (d. 1232).

Figure 7.18. Gephi: Scotland proper, ten or more co-witnessing acts

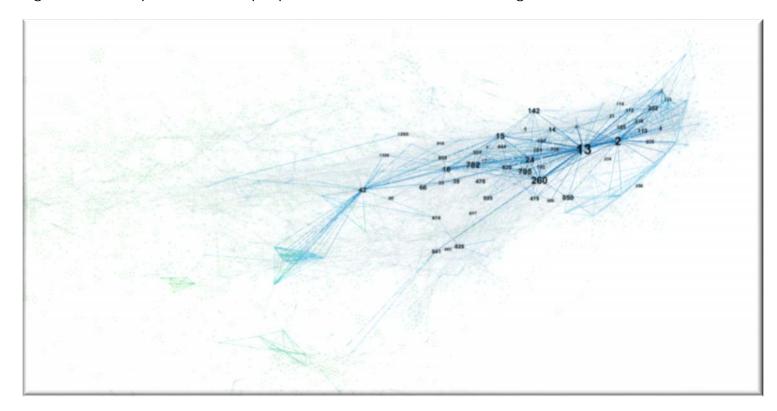


Figure 7.19. Gephi: Scotland proper, twenty or more co-witnessing acts

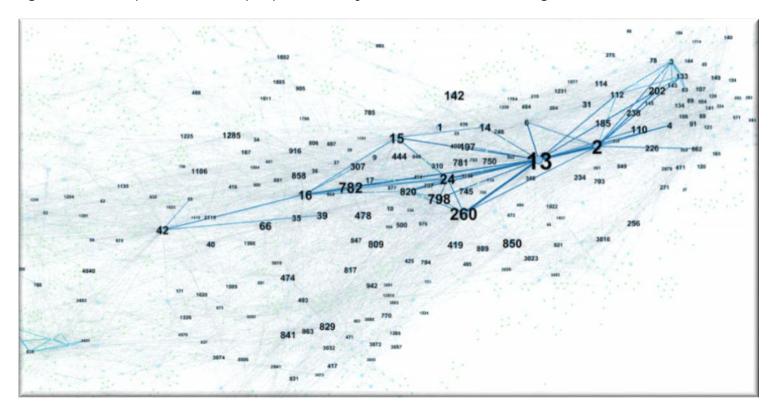


Figure 7.20. Gephi: Scotland proper, thirty or more co-witnessing acts

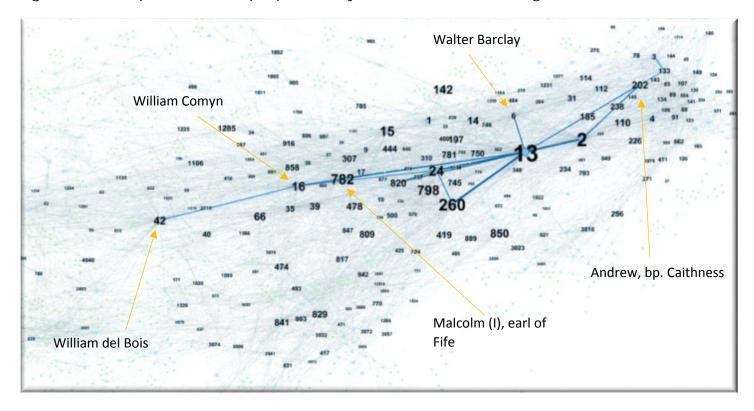


Figure 7.21. Gephi: Scotland proper, forty or more co-witnessing acts

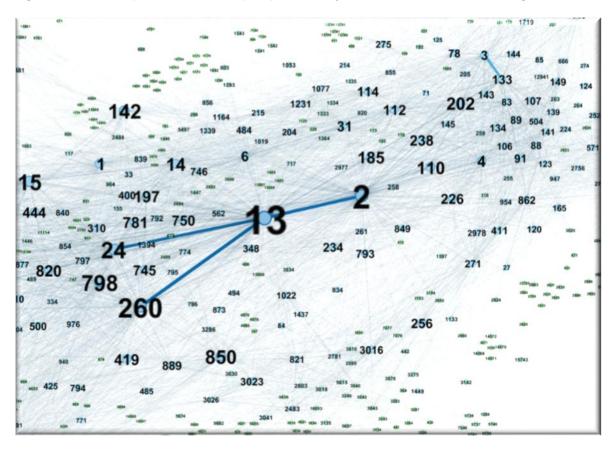
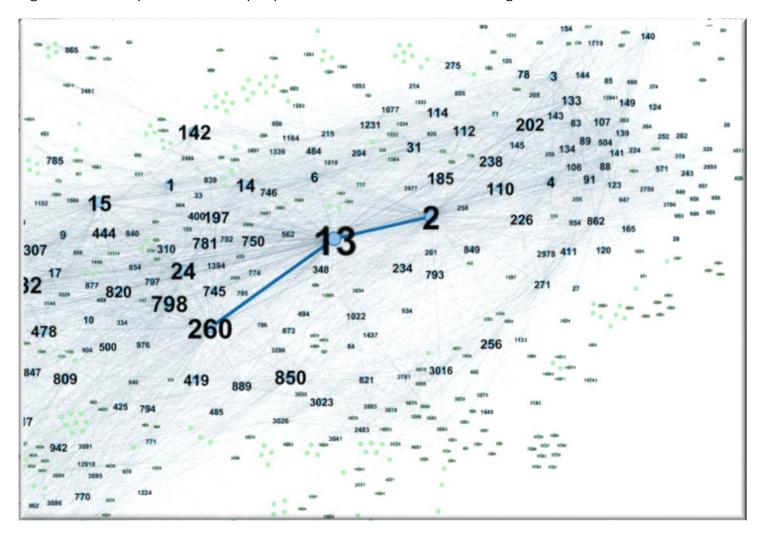


Figure 7.22. Gephi. Scotland proper, 55 or more co-witnessing acts



Scotland proper dataset, witnesses to royal documents (H1)

There were 534 documents in the study of royal charters relating to Scotland proper, of which 520 were charters, 6 were charter/brieves, five were notifications, and two were settlements. There were 681 witnesses in the study, two of whom were women, and there were 9782 edges connecting them up.

Table 7.9. Centrality: Top 20 Witnesses by Degree

Poms ID	Person	Degree	Betweenness Centrality	Eigenvector Centrality
13	Duncan (II), earl of Fife (d.1204)	248	19083.3	1
2	Matthew, bishop of Aberdeen (d.1199)	210	15746.31	0.902159
15	Philip de Valognes, chamberlain (d.1215)	174	14355.41	0.755704
16	William Comyn, earl of Buchan (d.1233)	173	23789.55	0.623858
24	William Hay (I), lord of Errol (d.c.1201)	173	5202.116	0.796621
202	Andrew, bishop of Caithness (d.1184)	170	8882.087	0.741165
3	Walter Stewart (I), son of Alan (d.1177)	165	5020.404	0.680665
260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	159	4901.973	0.770658
14	Robert de Quincy (d.1200)	152	3169.89	0.750835
112	Richard de Moreville (d.1189 or 1190)	146	4471.07	0.696345
798	Richard de Prebenda, bishop of Dunkeld (d.1210)	142	3020.392	0.680882
110	Gilla Brigte, earl of Angus (d.×1189)	141	2575.092	0.722699
133	Nicholas of Roxburgh, chancellor (d.1171?)	135	2794.402	0.597034
782	Malcolm (I), earl of Fife (d.1229)	133	8937.892	0.574969
4	Ness, son of William, lord of Leuchars (d.1178×83)	130	1764.723	0.607796
78	Walter de Bidun (d.1178)	122	4155.429	0.541806
1357	Walter Comyn, earl of Menteith (d.1258)	120	14899.54	0.200229
6	Walter Barclay, chamberlain (d.c.1193)	115	1387.588	0.61342
238	Malcolm, earl of Atholl (d.c.1197)	114	1283.844	0.629891
307	Robert of London (d.1225)	114	6930.382	0.500855

Table 7.9 reveals the top 20 witnesses by degree. As with the study of the whole kingdom (see Table 5.9), Duncan (II), earl of Fife (d. 1204) had the highest degree and eigenvector centrality. Indeed, his degree here (248) is only not much smaller than his degree in the larger dataset (277). The steward Walter son of Alan (I) (d. 1177), second in the degree table of all royal charters, falls to seventh place among the Scotland proper H1 documents. Andrew, bishop of Caithness (d. 1184), moves up from tenth place in Figure 5.9 to sixth place here. Richard de Moreville (d. 1189/90) fell from sixth place to tenth place. For the most part, however, actors like Matthew, bishop of Aberdeen; Philip de Valognes; William Comyn, earl of Buchan; William Hay; and Robert de Quincy occupied comparable or similar positions of importance in both networks. It is also useful to remark on the similarity between this dataset and the study of all Scotland proper witnesses, above. Earl Duncan and Bishop Matthew occupied the top two slots in terms of degree and eigenvector there as well.

In terms of the most productive co-witnessing 'relationships', Earl Duncan (II)'s unassailable position as a person of key significance is even more clearly obvious among Scotland proper royal documents than when look at SP documents more generally. Earl Duncan was part of eight of the pairs among the 19 involving 25 or more co-witnessing acts. From the previous generation, Walter Stewart (I) (d. 1177) witnessed 44 documents with the chancellor Nicholas (d. 1171). From the generation after Earl Duncan (II), William Comyn, earl of Buchan (d.1233) witnessed 28 times with William del Bois, chancellor (d.1232). This is the 'latest relationship' in time of all the pairs who witnessed together more than 20 times.

Table 7.10. Most productive co-witnessing 'relationships' (SP/H1)

Person 1	Person 2	#docs
Duncan (II), earl of Fife (d.1204)	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	49
Duncan (II), earl of Fife (d.1204)	William Hay (I), lord of Errol (d.c.1201)	46
Walter Stewart (I), son of Alan (d.1177)	Nicholas of Roxburgh, chancellor (d.1171?)	44
Duncan (II), earl of Fife (d.1204)	Matthew, bishop of Aberdeen (d.1199)	40
William Hay (I), lord of Errol (d.c.1201)	Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	32
Duncan (II), earl of Fife (d.1204)	Walter Barclay, chamberlain (d.c.1193)	29
William Comyn, earl of Buchan (d.1233)	William del Bois, chancellor (d.1232)	28
William Hay (I), lord of Errol (d.c.1201)	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	28
William Comyn, earl of Buchan (d.1233)	William Hay (I), lord of Errol (d.c.1201)	27
Richard de Moreville (d.1189 or 1190)	Walter Stewart (I), son of Alan (d.1177)	27
Matthew, bishop of Aberdeen (d.1199)	Andrew, bishop of Caithness (d.1184)	27
Nicholas of Roxburgh, chancellor (d.1171?)	Andrew, bishop of Caithness (d.1184)	27

Duncan (II), earl of Fife (d.1204)	William Comyn, earl of Buchan (d.1233)	26
Duncan (II), earl of Fife (d.1204)	Richard de Moreville (d.1189 or 1190)	26
Duncan (II), earl of Fife (d.1204)	Andrew, bishop of Caithness (d.1184)	25
Duncan (II), earl of Fife (d.1204)	Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	25
Philip de Valognes, chamberlain (d.1215)	William Hay (I), lord of Errol (d.c.1201)	25
Matthew, bishop of Aberdeen (d.1199)	Walter Stewart (I), son of Alan (d.1177)	25
Walter Stewart (I), son of Alan (d.1177)	Ingram, bishop of Glasgow (d.1174)	25
Philip de Valognes, chamberlain (d.1215)	William Comyn, earl of Buchan (d.1233)	24
Walter Stewart (I), son of Alan (d.1177)	Andrew, bishop of Caithness (d.1184)	24
Nicholas of Roxburgh, chancellor (d.1171?)	David Oliphant (12C)	24
William Comyn, earl of Buchan (d.1233)	Malcolm (I), earl of Fife (d.1229)	23
Duncan (II), earl of Fife (d.1204)	Philip de Valognes, chamberlain (d.1215)	22
Matthew, bishop of Aberdeen (d.1199)	Ness, son of William, lord of Leuchars (d.1178×83)	22
Duncan (II), earl of Fife (d.1204)	Richard de Prebenda, bishop of Dunkeld (d.1210)	21
Duncan (II), earl of Fife (d.1204)	Malcolm (I), earl of Fife (d.1229)	21
Richard de Moreville (d.1189 or 1190)	Matthew, bishop of Aberdeen (d.1199)	21
Richard de Moreville (d.1189 or 1190)	Nicholas of Roxburgh, chancellor (d.1171?)	21
Matthew, bishop of Aberdeen (d.1199)	Nicholas of Roxburgh, chancellor (d.1171?)	21

Figure 7.23. Netdraw: witnesses to SP/H1 documents, >5 co-witnessing instances

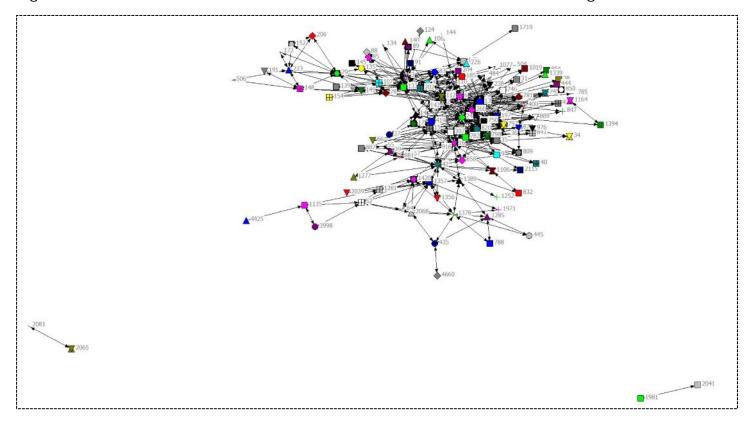


Figure 7.24. Netdraw: witnesses to SP/H1 documents, >10 co-witnessing instances

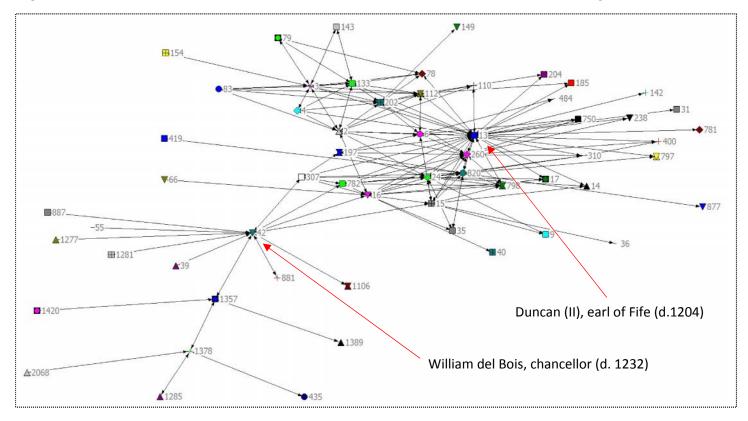


Figure 7.25. Netdraw: witnesses to SP/H1 documents, >20 co-witnessing instances

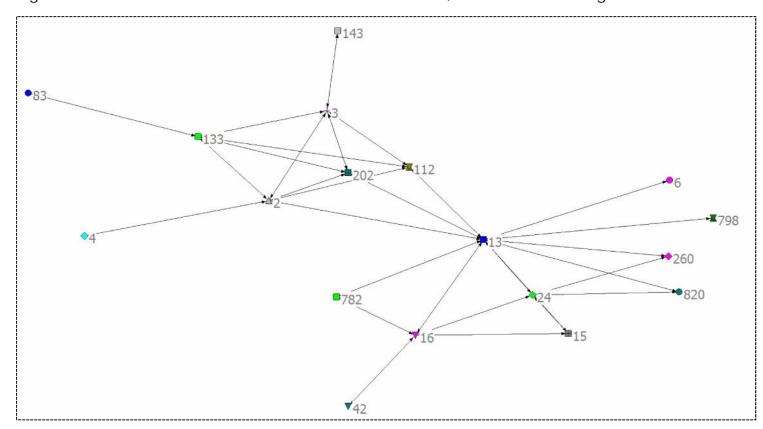


Figure 7.26. Netdraw: witnesses to SP/H1 documents, >25 co-witnessing instances

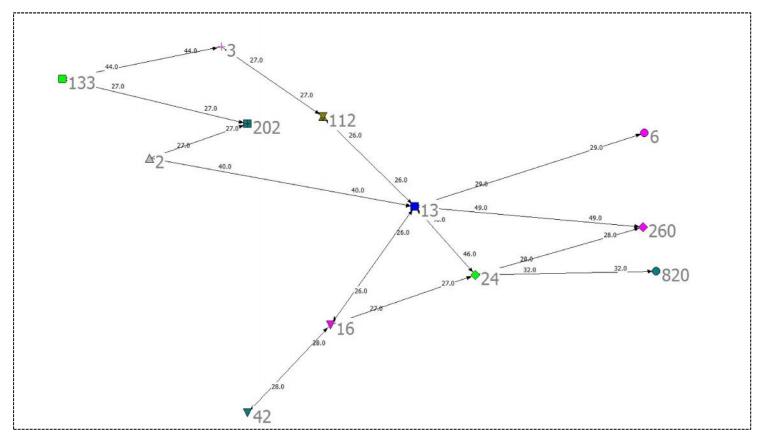
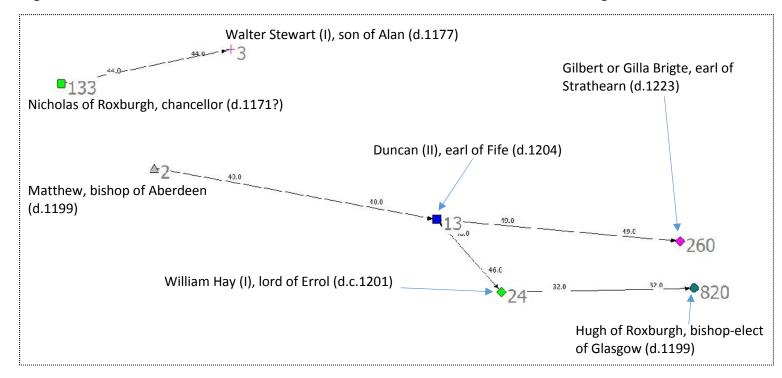


Figure 7.27. Netdraw: witnesses to SP/H1 documents, >30 co-witnessing instances



Scotland proper dataset, witnesses to ecclesiastical documents (H2)

The Social Network Analysis of all ecclesiastical documents (H2) in the Scotland proper dataset includes 343 documents, of which 338 are charters and 5 are notifications. The study comprises 1065 witnesses, none of whom were women, and 10845 edges.

The table of centrality figures shows the predominance of the episcopal church of St Andrews, and as such bears comparison with the study of all witnesses to H2 documents in the previous chapter. Indeed, Matthew, bishop of Aberdeen (d. 1199) had the highest centrality in terms of degree and eigenvector, just as in the larger study. The position of Duncan (II), earl of Fife (d. 1204), was more central in the Scotland proper study than the study for the whole kingdom, in that he moved from seventh place in degree up to second place, and from third in eigenvector (86.7%) up to second place (94.9%). While the vast majority of the most central players in this study were personnel of the church of St Andrews, it is also noteworthy that archdeacons of Aberdeen and Dunkeld appear below.

Table 7.11. Centrality: Top 20 witnesses by degree

PoMS ID	Person	Degree	Between	Eigenvector
2	Matthew, bishop of Aberdeen (d.1199)	190	41453.66	1
829	Ranulf de Wat, archdeacon of St Andrews (d.1209)	150	29726.78	0.726505
13	Duncan (II), earl of Fife (d.1204)	149	29742.44	0.949179
3016	Alexander, chaplain of bishops of St Andrews (12C)	134	17691.96	0.698584
411	Andrew, archdeacon of Lothian (fl.1147×59-1178×84)	119	3888.976	0.787905
862	Herbert Scott, master, clerk (fl.1144×59-1172×78)	116	2753.319	0.729274
850	John Scott, bishop of Dunkeld (d.1203)	113	10548.01	0.718644
271	Robert, son of Saewulf, bishop's chancellor	107	2697.556	0.712188
493	John of Leicester, bishop of Dunkeld (d.1214)	106	9549.439	0.563483
865	Simon, archdeacon of Aberdeen (fl.1189×96-1202×03)	101	13934.76	0.513235
863	Isaac Scott, master, clerk	96	6769.027	0.478635
2762	Henry, archdeacon of Dunkeld (fl.1183×1203-1220×25)	96	23116.89	0.132771
165	Aiulf, dean of Lothian (fl.1150/51-1186)	94	1623.319	0.652766
821	Walter of Roxburgh, archdeacon of St Andrews (fl.1165×72-1179×88)	92	10813.16	0.469978
770	William of Hailes, master, dean of St Andrews (fl.1189×98)	92	5233.508	0.543874
2483	Gamel, doorward, master (St Andrews)	92	7895.863	0.519511
1022	Odo of Kinninmonth, steward, marischal (d.c.1195)	91	4224.169	0.603626
256	Walter, prior of St Andrews (fl.1160-1198×99)	91	5618.928	0.595782
474	Guy, abbot of Lindores (d.1219)	90	12625.5	0.440191
2978	Stephen, clerk (St Andrews)	88	1730.24	0.678106

The most productive co-witnessing 'relationships' in the study of witnesses to H2 documents in the Scotland proper dataset were also representative of the importance of the church of St Andrews and the production and survival of episcopal documents there. As such, it bears much resemblance to the equivalent chart among witnesses for the whole kingdom. Much of the diocese of St Andrews was south of the Firth of Forth, however, and charters of the bishops of St Andrews dealing with the southern part of the diocese were not included in the Scotland proper dataset, which is responsible for the differences between the two studies. The archdeacon of St Andrews Laurence of Thornton appears as much more significant in this study, and he was involved in six of the pairs in the table. Indeed, the time of Bishop William Malveisin (1202-38) is better represented in this dataset, in relative terms, with figures like Peter the chaplain and clerk, Master Michael, the chaplain and clerk, Simon de Noisy the clerk, and Master Stephen of Lilliesleaf registering in more prominent positions.

Table 7.12. Most productive co-witnessing 'relationships' (SP/H2)

Person1	Person2	#docs
Matthew, bishop of Aberdeen (d.1199)	Herbert Scott, master, clerk (fl.1144x59-1172x78)	24
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Peter, chaplain and clerk of Bishop Malveisin	21
Michael, master, clerk, chaplain (fl.1201-1220x25)	Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	20
Michael, master, clerk, chaplain (fl.1201-1220x25)	Peter, chaplain and clerk of Bishop Malveisin	20
Peter, chaplain and clerk of Bishop Malveisin	Stephen of Lilliesleaf, master, clerk, persona	20
Walter of Roxburgh, archdeacon of St Andrews (fl.1165x72-1179x88)	Alexander, chaplain of bishops of St Andrews (12C)	19
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Edward Murray, master, canon, bishop's clerk	19
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Simon de Noisy, clerk of Bishop William of St Andrews	19
Robert, son of Saewulf, bishop's chancellor	Alexander, chaplain of bishops of St Andrews (12C)	18
Michael, master, clerk, chaplain (fl.1201-1220x25)	Stephen of Lilliesleaf, master, clerk, persona	18
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	Stephen of Lilliesleaf, master, clerk, persona	18
Laurence of Thornton, archdeacon of St Andrews (d.1238x40)	William of Gullane, rector of Gullane	18
Peter, chaplain and clerk of Bishop Malveisin	Edward Murray, master, canon, bishop's clerk	18
William of Gullane, rector of Gullane	Simon de Noisy, clerk of Bishop William of St Andrews	18

Figure 7.28. Netdraw: witnesses to SP/H2 documents, >5 co-witnessing instances

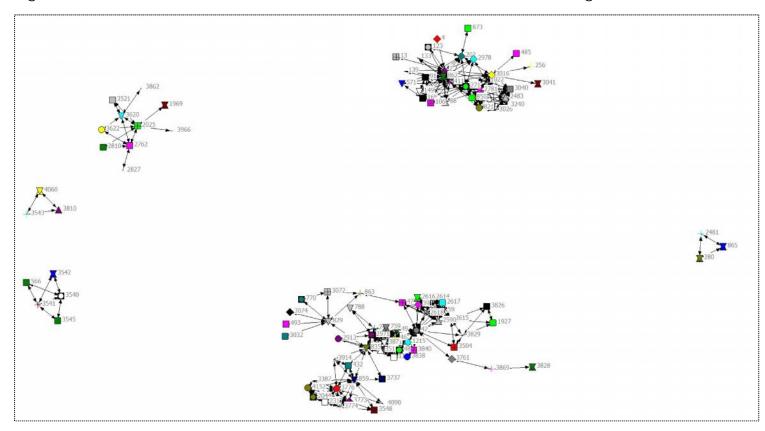


Figure 7.29. Netdraw: witnesses to SP/H2 documents, >10 co-witnessing instances

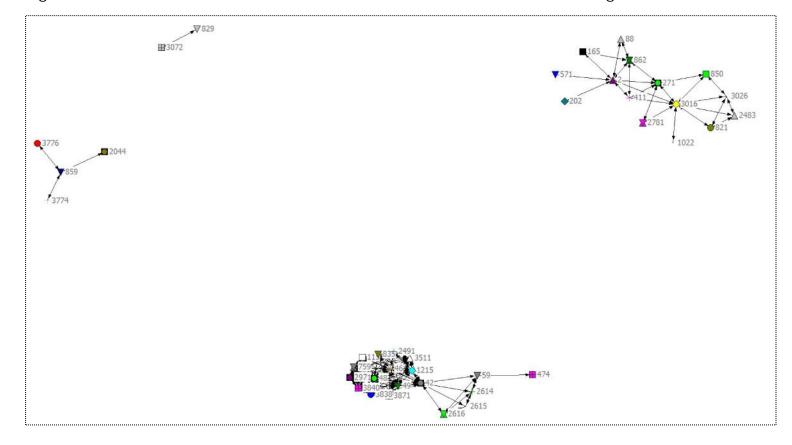


Figure 7.30. Netdraw: witnesses to SP/H2 documents, >15 co-witnessing instances

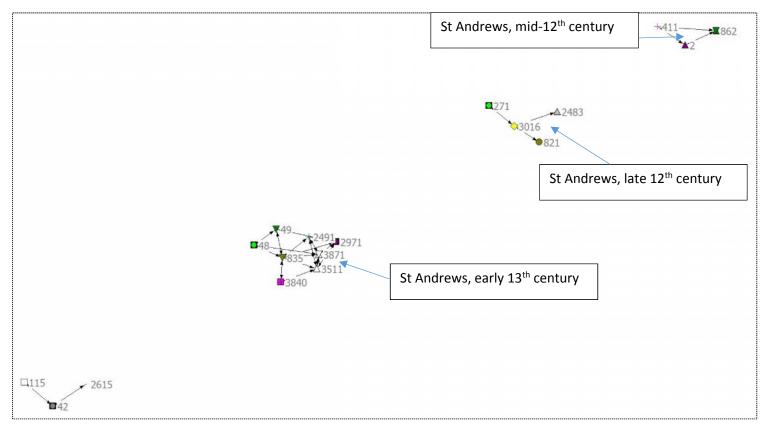
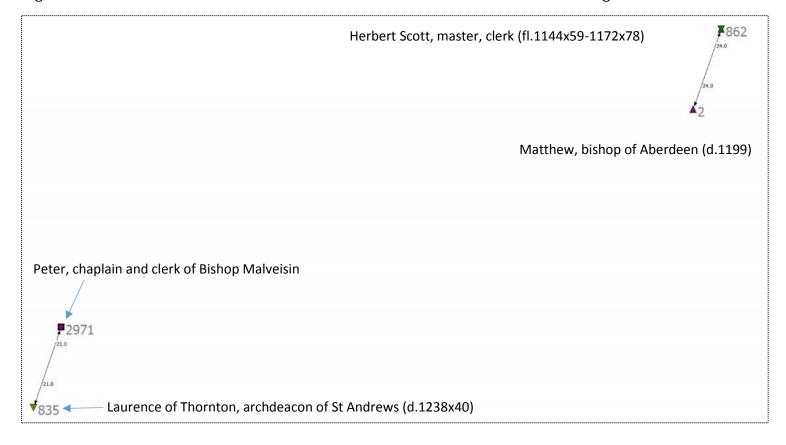


Figure 7.31. Netdraw: witnesses to SP/H2 documents, >20 co-witnessing instances



The sociograms of the Scotland proper study of H2 witnesses also bear comparison to the graphs of all H2 witnesses for the whole kingdom. We see the existence of separate segments of witnesses at relatively low levels of co-witnessing, and at the level of more than 15 co-witnessing instances, nearly all of the witnesses in the network were attached to the church of St Andrews, with the divisions between the segments being based on three distinct chronological periods, from the mid-twelfth century through to the mid-thirteenth century. Representing the later period, the archdeacon Laurence of Thornton co-witnessed with Peter the chaplain and clerk 21 times, and representing the earlier period, Matthew, bishop of Aberdeen, and archdeacon of St Andrews from ca 1150 to 1172, co-witnessed with Master Herbert Scot, the clerk, a total of 24 times.

Scotland proper dataset, witnesses to lay or private documents (H3)

The study of lay or private charters (H3) in the Scotland proper dataset comprises 613 documents, only about a third of all such documents in the database. Of these, 608 were charters, 2 were charter/brieves, and three were notifications. There were 2626 witnesses to these documents, and 22,322 edges in the social network. Of the witnesses, 18 were women.

Table 7.13. Centrality: top 25 witnesses by degree

PoMS ID	Person	Degree	Betweenness	Eigenvector
			Centrality	Centrality
13	Duncan (II), earl of Fife (d.1204)	228	211635.7	1
782	Malcolm (I), earl of Fife (d.1229)	153	95702.18	0.797241
42	William del Bois, chancellor (d.1232)	152	147663.5	0.559057
1802	William Wascelin, knight	133	62031.37	0.587609
66	David Hay, lord of Errol (d.1237×41)	130	116738.6	0.569724
5330	Henry, son of Geoffrey de Liberatione of Perth	125	154566.6	0.469267
2	Matthew, bishop of Aberdeen (d.1199)	124	37646.57	0.558863
64	Henry of Stirling, son of Earl David	122	85404.83	0.35613
15	Philip de Valognes, chamberlain (d.1215)	121	45776.25	0.702431
40	William Malveisin, bishop of St Andrews (d.1238)	120	136587	0.496282
1389	John Hay (I), lord of Naughton (d.×Oct.1266)	120	167251.9	0.340937
3023	Adam of Kilconquhar, brother of Earl Duncan (father of Duncan)	120	43096.87	0.452026
1285	Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	115	73481.37	0.607322
260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	114	45637.14	0.683936

2067	Gilbert Hay (I), lord of Errol (d.1263) (son of David)	113	134957.6	0.257294
16	William Comyn, earl of Buchan (d.1233)	112	32608.14	0.734006
1233	Philip Melville, justiciar of Scotia	111	101927.4	0.342594
5364	John Cameron, sheriff of Perth	108	120904.4	0.198633
142	David, earl of Huntingdon (d.1219)	106	47010.21	0.48502
2190	Robert Mowat, knight, justiciar, sheriff of Forfar	105	116138	0.190034
1326	Duncan, son of Earl Duncan (II) of Fife	100	41428.49	0.416404
1805	Walkelin, son of Stephen	97	24009.76	0.503629
1	William I, king of Scots (d.1214)	95	19142.03	0.570169
1981	Alexander Comyn, earl of Buchan (d.1289)	90	72429.12	0.192259
5808	Ralph de Lascelles, knight	90	80618.15	0.145053

As in the study of witnesses to all H3 documents, Duncan (II), earl of Fife (d.1204) has the highest centrality in all three measurements – degree, betweenness, and centrality. His position among Scotland proper documents is unassailable. With 228 contacts, he had 75 more than the person with the next highest degree, his son Malcolm (I), earl of Fife (d.1229). Earl Malcolm also comes in the second slot in eigenvector, but his score is only 79.7% that of his father. The third, fourth, and fifth slots according to eigenvector were occupied by William Comyn, earl of Buchan (d.1233), Philip de Valognes, chamberlain (d.1215), and Gilbert or Gilla Brigte, earl of Strathearn (d.1223), respectively. All of these men were prominent actors in the networks of royal charter witnesses, as explored in chapter 5, above. Of the most central witnesses, some can be associated with [142] David earl of Huntingdon, including his household knight, [1802] William Wascelin, his son [64] Henry of Stirling, and his vassal [1805] Walkelin son of Stephen. Others were associated with the aforementioned earls of Fife, namely Earl Duncan's brother [3023] Adam of Kilconquhar and Earl Duncan's son [1326] Duncan. The Hay family were also prominent figures, particularly [66] David, [1389] John, and [2067] Gilbert. Finally, the prominence of a number of justiciars and sheriffs is noteworthy.

Table 7.14. Most productive co-witnessing 'relationships' (SP/H3)

Person 1	Person 2	#docs
Abraham, bishop of Dunblane (fl.1210x14-1220x25)	Malise, son of Gilla na Naem, steward of earls of Strathearn	17
Abraham, bishop of Dunblane (fl.1210x14-1220x25)	Brice, persona of Crieff	14
Robert, earl of Strathearn (1223-45)	Malise, son of Gilla na Naem, steward of earls of Strathearn	12
Abraham, bishop of Dunblane (fl.1210x14-1220x25)	Gilbert, archdeacon of Dunblane (fl.1203x10-1235x39)	12
Malise, son of Gilla na Naem, steward of earls of Strathearn	Brice, persona of Crieff	12
William Wascelin, knight	Walkelin, son of Stephen	11
Robert, earl of Strathearn (1223-45)	Abraham, bishop of Dunblane (fl.1210x14-1220x25)	11
Malise, son of Gilla na Naem, steward of earls of Strathearn	Gilbert, archdeacon of Dunblane (fl.1203x10-1235x39)	11
Malise, son of Ferteth earl of Strathearn (d.a.1214)	Gilla na Naem, steward of Earl Gilbert	10
Malise, son of Ferteth earl of Strathearn (d.a.1214)	Constantine, judex (PER)	10
Fergus, son of Gilbert, earl of Strathearn (d.c.1247)	Abraham, bishop of Dunblane (fl.1210x14-1220x25)	10
Malise, son of Gilla na Naem, steward of earls of Strathearn	Malise, son of Earl Gilbert of Strathearn (d.c.1272)	10

As we saw in the previous chapter, the Coldingham corpus of documents caused the H3 study to be dominated by minor landholders from Berwickshire, at least in terms of those who witnessed most frequently. These charters, of course, do not appear in the Scotland proper dataset. As Table 7.14 demonstrates, the richest seam of lay charters north of Forth deal with the province and earldom of Strathearn. As such, the points to be made here will resemble closely the case study of the charters of the earls of Strathearn laid out in the previous chapter. Bishop Abraham and Archdeacon Gilbert of Dunblane appear here, along with various close relatives and stewards of the earls of Strathearn. The only exception to this is the fact that William Wascelin and Walkelin son of Stephen co-witnessed 11 times. This should be seen in the context of the charters of David earl of Huntingdon.

Figure 7.32. Netdraw: witnesses to SP/H3 documents, >3 co-witnessing instances

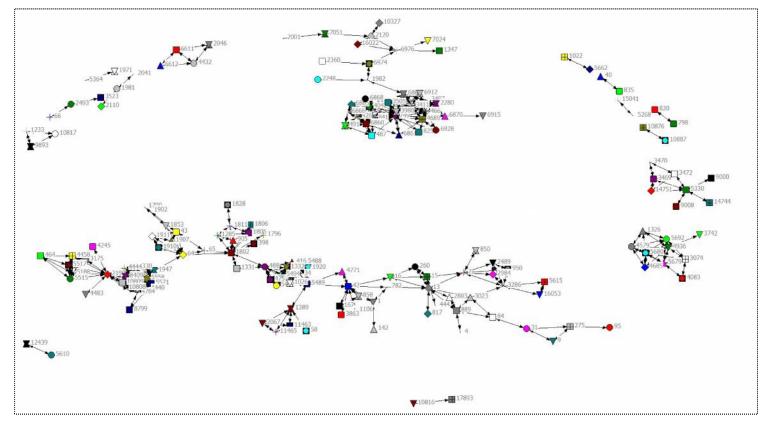
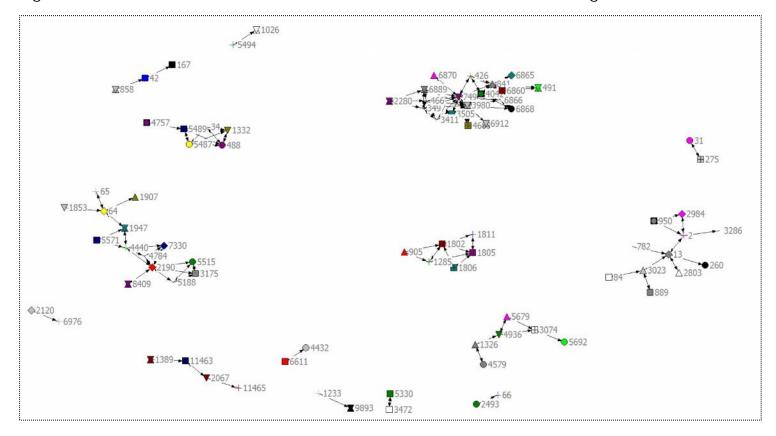


Figure 7.33. Netdraw: witnesses to SP/H3 documents, >5 co-witnessing instances



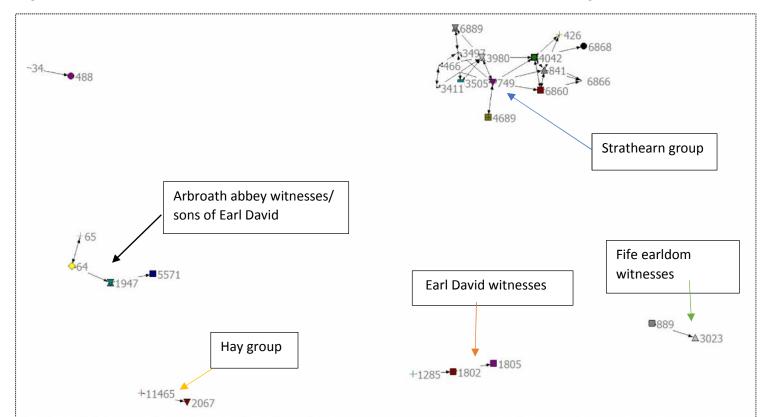
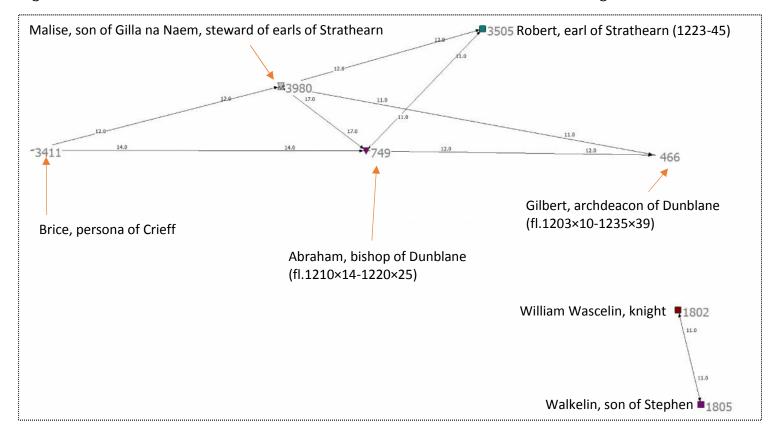


Figure 7.34. Netdraw: witnesses to SP/H3 documents, >7 co-witnessing instances

The social networks of witnesses to H3 documents in the Scotland proper dataset are fairly thoroughly interconnected at the level of more than three co-witnessing acts, but break down into multiple segments by the level of more than five instances. As Figure 7.34 shows, the segment associated with the earldom of Strathearn is the most populous, but there are still a few other groups comprising mostly people associated with David earl of Huntingdon, the earls of Fife, or the Hay family. Figure 7.34 shows the seven individuals who witnessed together more than 10 times. This includes three triads, including five individuals were active in Strathearn in the early to mid-thirteenth century, as well as a dyad of two vassals of Earl David.

Figure 7.35. Netdraw: witnesses to SP/H3 documents, >10 co-witnessing instances



8 Ego-Networks

Ego-networks are networks that are defined by a single cenral individual, known as ego. Everyone in the network is connected to ego. The ego-network includes ties between these other actors, known as alters, and ego, as well as ties between the alters themselves. One of the main analytical advantages of ego-networks is being able to ask how many of the people whom ego knows also know each other, in the stated context. While it is possible to construct an ego-network specially 'from scratch', all of the ego-networks we examine here are sub-sets of 'whole networks'. Whole networks are defined by factors that are extraneous to ego; in our case, they are defined by bodies of historical documents. The examples which follow are ego-networks derived from the larger whole network of witnesses to documents in the PoMS database, 1093 to 1286, of the five specified document types: charters, charter/brieves, notifications, agreements and settlements. All of the people who have co-witnessed alongside the nominated 'ego' will appear in the ego-network; thus, the size of the ego-network is always the same as the degree centrality of ego in the whole network.

Ego-network size and density

One feature of the ego-network that is perhaps most commonly explored is density. The density of an ego-network is measured as the percentage of potential ties in a network which are actualised, or, in other words, the extent to which alters are connected. A network in which all of ego's friends also know each other, all of the potential ties are actualised, and consequently the density is 1 (or 100 percent).

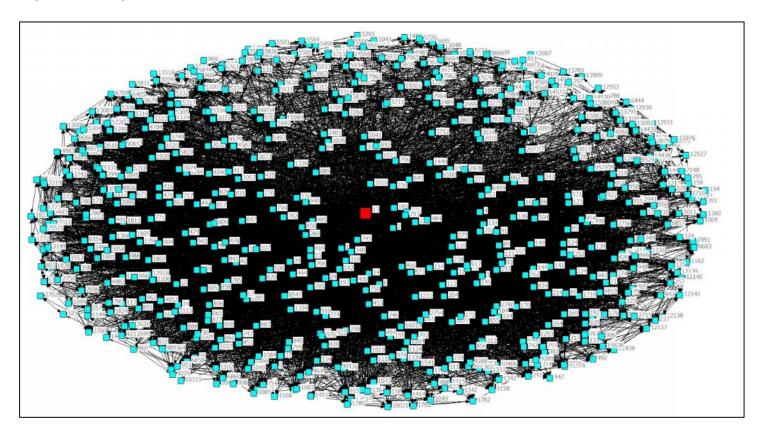
A useful place to start may be to examine the density of the ego-networks of PoMS actors who have been identified as important through high centrality. The following table examines the densities of the ego-networks of the fifteen individuals with the highest degree centrality in the whole network. In other words, it lists the top 15 by size of ego-network.

Table 8.1. Top fifteen players by size of ego-network, with densities

Poms ID	Name	Degree/ size	Egonet Density
13	Duncan (II), earl of Fife (d.1204)	585	8.4
42	William del Bois, chancellor (d.1232)	476	8.04
2	Matthew, bishop of Aberdeen (d.1199)	475	10.4
142	David, earl of Huntingdon (d.1219)	411	13.12
858	Walter of St Albans, bishop of Glasgow (d.1232)	380	10.58
40	William Malveisin, bishop of St Andrews (d.1238)	379	10.82
782	Malcolm (I), earl of Fife (d.1229)	377	11.54
15	Philip de Valognes, chamberlain (d.1215)	363	13.57
745	Jocelin, bishop of Glasgow (d.1199)	356	12.06
260	Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	354	13.81
798	Richard de Prebenda, bishop of Dunkeld (d.1210)	347	15.39
444	Patrick (I), earl of Dunbar (d.1232)	343	12.56
850	John Scott, bishop of Dunkeld (d.1203)	337	13.34
1285	Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	327	11.14
1	William I, king of Scots (d.1214)	321	16.56

All of the individuals listed in the above table have ego-networks of more than 320 people, and all have ego-network densities of less than 17 percent. This corresponds to a general principle that the larger the ego-network, the less likely that all of one's contacts will also be connected to each other. If charter witnessing were a proxy for knowing one's contemporaries, which of course is a complicated question, then only about 8% of Earl Duncan II of Fife's will have known each other, which would raise the likelihood that Earl Duncan acted as a bridge or conduit between those other actors. Of course, the charters witnessed by Earl Duncan were spread out across nearly fifty years, meaning that in reality, some of Earl Duncan's alters would not have even been alive at the same time.

Figure 8.1. Ego-network of Earl Duncan II of Fife



As we can see from Table 8.1, the fifteen largest ego-networks all have densities between about 8 and 17; indeed, most of them are below about 13. These are among the lowest-density ego-networks of PoMS actors, and this makes sense, because the larger number of contacts in their ego-networks are mathematically less likely to be connected to each other as compared to contacts in much smaller networks, where it is easier to come closer to 'completion' (100% density). How meaningful a measure can ego-network density be, though, if it is merely a reflection of how many people one with whom ego has co-witnessed, given the variations in documentary production discussed in previous chapters? How close is the correlation between degree or ego-network size and density? John Scott notes that 'the dependence of density on the size of a graph' does constitute a problem for comparing networks of different sizes, but mentions a countervailing trend to that already mentioned, whereby the number of contacts that ego can sustain tends to 'decline as the size of the network increases'. This is due to real constraints on time available for meaningful human interaction (Scott 2000, 74-5). We must remember, however, that our historical networks reflect chronological spans of time, which must be kept in mind during historical network analysis.

Table 8.2. Fifteen actors with lowest ego-network densities

Poms ID	Name	Egonet Density	Degree/ size
3350	Adam of Makerstoun, master, provost (d.1280×86)	7.99	155
42	William del Bois, chancellor (d.1232)	8.04	476
13	Duncan (II) earl of Fife (d. 1204)	8.4	585
2190	Robert Mowat, knight, justiciar, sheriff of Forfar	9.43	153
2762	Henry, archdeacon of Dunkeld (fl.1183×1203-1220×25)	9.69	175
5364	John Cameron, sheriff of Perth	9.76	136
43	John, abbot of Lindores (fl.1219-44)	10.05	159
788	Andrew Murray, bishop of Moray (d.1242)	10.17	273
4427	William, son of Earl Patrick (I) (d.1253)	10.29	140
1378	Walter Stewart (II), son of Alan (d.1241)	10.3	253
2	Matthew, bishop of Aberdeen (d.1199)	10.4	473
858	Walter of St Albans, bishop of Glasgow (d.1232)	10.58	380
2067	Gilbert Hay (I), lord of Errol (d.1263) (son of David)	10.7	137
3432	Thomas Crook, knight	10.79	110
40	William Malveisin, bishop of St Andrews (d.1238)	10.82	379

Table 8.2 shows the lowest ego-network densities when we analyse the ego-networks of the 315 individuals in the whole-network study of the PoMS database with degrees of 100 or above. Of the fifteen individuals with the lowest ego-net densities, ten actors had degrees/ ego-net sizes less than 320 (in boldface) and were thus not in the table of the fifteen players with the largest ego-networks. In other words, of the actors with 100 co-witnessing contacts or more, one third of the fifteen individuals with the lowest densities had more than 320 contacts, and two-thirds had between 100 and 320 contacts. Indeed, over half had ego-network sizes of between 100 and 200, and three of the "top five" had degrees below 200. Thus, it is clear that while many of the largest ego-networks were relatively less dense, at the same time, many of the least dense ego-networks were much smaller. Presumably, if we had the time to analyse all of the PoMS actors with fewer than 100 contacts, we would find some ego-networks with low densities. The significance of low-density networks will be explored below.

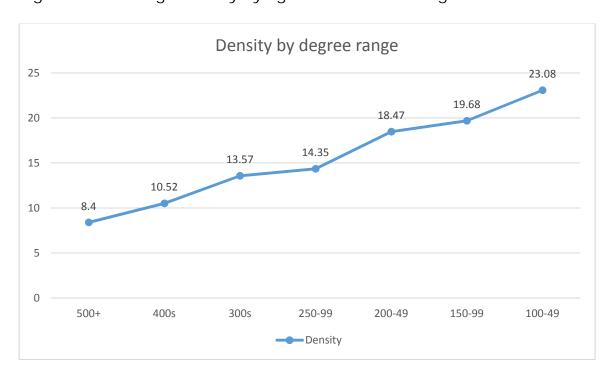


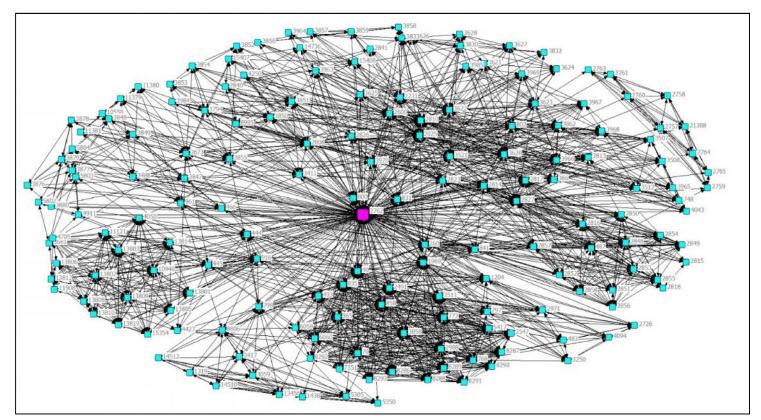
Figure 8.2. Average density by ego-network size ranges

As Figure 8.2 shows, there is still a general tendency for the smaller ego-networks to be denser. The average density of ego-networks where ego had between 400 and 500 co-witnessing contacts was only 10.52, while the average density of those networks with between 100 and 149 contacts was more than twice that, at 23.08. Nevertheless, as presence of low-density networks of that size reveal, this is only a general trend and not an exact correlation. Therefore, it is still meaningful to compare ego-network densities.

The following sociograms illustrate the ego-networks of various PoMS actors with a range of ego-network sizes and densities. Figure 8.3 is a sociogram of the ego-network of Henry, archdeacon of Dunkeld [PoMS, no. 2762]. This is an example of a small network – Henry witnessed alongside 175 other actors in a total of 26 documents included in the study. Nevertheless, he has a remarkably low ego-net density of 9.69. It is visible in the sociogram that many of the nodes appear in groups of higher density, but that these are not particularly well connected to each other. Figure 8.4, on the other hand, serves as an illustration of a small network with high density. King David I (1124-53) [130] only witnessed alongside 25 other actors in only four included documents, which is not surprising given the early date of his reign in terms of the chronology of charter production and the fact that kings tended not to witness documents as often as did their chief advisors. King David's charter witnessing ego-

network is highly connected: its density of 57.33 means that over 57 percent of David's contacts also witnessed alongside one another. Figure 8.5 shows a very small ego-network with a very high density, that of Gilchrist mac inien (son of the daughter of) Samuel [no. 920]. In this network of 28 alters, 85.45% of them were connected to each other. It could be said that Gilchrist is deeply embedded in this very dense network. A network with 100% density is said to have reached 'completion' and would comprise a large clique.

Figure 8.3. Low density, small size: Henry, archdeacon of Dunkeld (fl.1183×1203-1220×25)



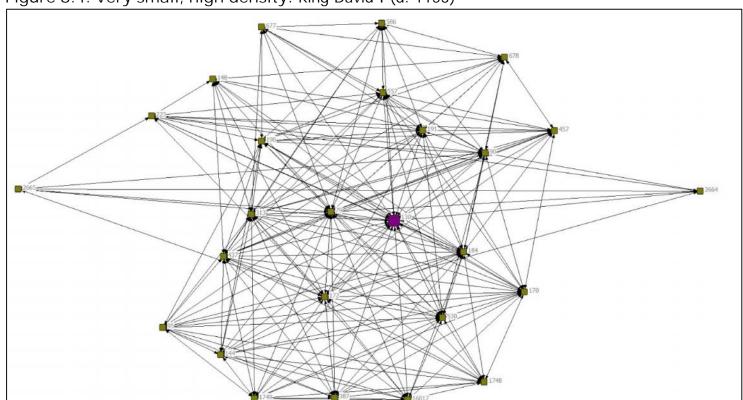
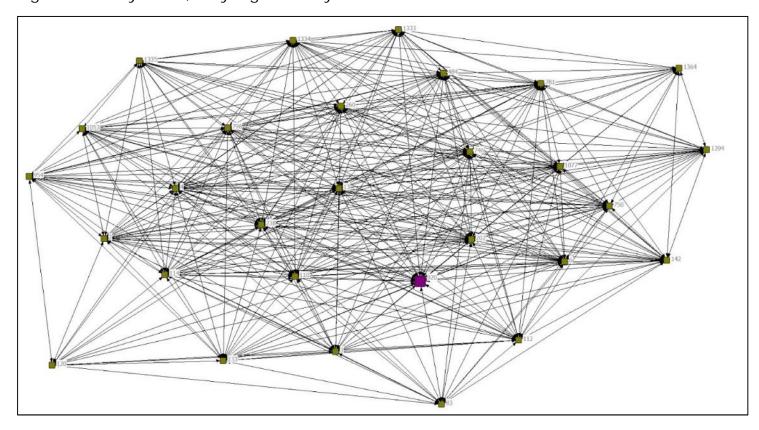


Figure 8.4. Very small, high density: King David I (d. 1153)

Figure 8.5 Very small, very high density: Gilchrist mac inien Samuel



Betweenness and Density

There is a certain parallel between betweenness centrality and density, given that both concepts focus on the position of an actor within the network structure and the relative advantages this position affords him or her. We may remember that William del Bois, chancellor (d. 1232) [42] had the highest betweenness centrality in the whole graph; thus, it should not be surprising that he also has the second-lowest ego-net density yet found in the PoMS database, at 8.04. For reasons explained by Christina Prell, however, the betweenness centrality of an actor in a whole network bears a complicated relationship to that actors position in his/her ego-network (Prell 2012, p. 124-5). However, ego will have a different betweenness centrality score in the ego-network than in the whole network, and that score is more straightforwardly related to the ego-net density.

Table 8.3: Betweenness Centrality – actors with 20 lowest ego-net densities

Person Name	ID	Ego-betweenness	Degree/ ego- net size	Ego-net density
Adam of Makerstoun, master, provost (d.1280×86)	3350	18747.30	155	7.99
William del Bois, chancellor (d.1232)	42	107525.03	476	8.04
Duncan (II) earl of Fife (d. 1204)	13	137516.64	585	8.4
Robert Mowat, knight, justiciar, sheriff of Forfar	2190	15641.31	153	9.43
Henry, adcn. Dunkeld (fl.1183×1203-1220×25)	2762	21763.12	175	9.69
John Cameron, sheriff of Perth	5364	12630.01	136	9.76
John, abbot of Lindores (fl.1219-44)	43	16310.76	159	10.05
Andrew Murray, bishop of Moray (d.1242)	788	40847.69	273	10.17
William, son of Earl Patrick (I) (d.1253)	4427	12743.03	140	10.29
Walter Stewart (II), son of Alan (d.1241)	1378	33797.48	253	10.3
Matthew, bishop of Aberdeen (d.1199)	2	76630.54	473	10.4
Walter of St Albans, bishop of Glasgow (d.1232)	858	60825.98	380	10.58
Gilbert Hay (I), lord of Errol (d.1263) (son of David)	2067	10410.55	137	10.7
Thomas Crook, knight	3432	8278.65	110	10.79
William Malveisin, bishop of St Andrews (d.1238)	40	60948.97	379	10.82
Bernard of Hadden, sheriff of Roxburgh	880	25268.25	226	10.88
John Maxwell, chamberlain, sh. Roxburgh (d.1241)	1281	33509.98	277	10.89
Laurence of Thornton, adcn. St Andrews (d.1238×40)	835	26403.11	233	10.99
Alan of Harcarse, knight	5954	10189.57	121	11.05
Walter Oliphant, justiciar of Lothian (d.1242)	1285	47075.48	327	11.14

Table 8.4: Betweenness Centrality – actors with 15 highest ego-net sizes (degree)

Person Name	ID	Ego- betweenness	Degree/ ego- net size	Ego-net density
Duncan (II) earl of Fife (d. 1204)	13	137516.64	585	8.4
William del Bois, chancellor (d.1232)	42	107525.03	476	8.04
Matthew, bishop of Aberdeen (d.1199)	2	76630.54	473	10.4
David, earl of Huntingdon (d. 1219)	142	47409.30	409	13.12
Walter of St Albans, bishop of Glasgow (d.1232)	858	60825.98	380	10.58
William Malveisin, bishop of St Andrews (d.1238)	40	60948.97	379	10.82
Malcolm (I), earl of Fife (d.1229)	782	54044.90	377	11.54
Philip de Valognes, chamberlain (d.1215)	15	37298.79	363	13.57
Jocelin, bishop of Glasgow (d.1199)	745	49181.71	356	12.06
Gilbert, earl of Strathearn (d. 1223)	260	39261.70	354	13.81
Richard de Prebenda, bishop of Dunkeld (d.1210)	798	30088.01	347	15.39
Patrick (I), earl of Dunbar (d.1232)	444	46258.25	343	12.56
John Scott, bishop of Dunkeld (d.1203)	850	40658.70	337	13.34
Walter Oliphant, justiciar of Lothian (d.1242)	1285	47075.48	327	11.14
William I, king of Scots (d.1214)	1	26471.33	321	16.56

The above tables show that ego-betweenness is related to both size of the network and density. In general, the larger the network size, the higher betweenness number. This is also offset by the density of the actor, so that Walter Oliphant [1285], who has a degree of 327 and a density of 11.14, has a betweenness of about 47K. Meanwhile, David earl of Huntingdon [142] also has a betweenness score of about 47K, despite having a much higher degree of 409. This is because his density, at 13.12, is also considerably higher.

The conceptual world of high-density networks

As Charles Kadushin writes, 'in network terms, safety or supportive systems are usually equivalent to density in networks, a condition that has been generally associated with "social support", "cohesion" and "embeddedness".' (Kadushin, 60). Dense networks are often considered to engender situations of 'trust, cooperation, mutual support,' and a 'sense of solidarity and belonging' (Crossley, 31). These are underpinned by important work on social theory by sociologists like Robert Putnam, who elaborated the concept of 'bonding capital' as a kind of social capital, and of James Coleman, who explored the inherent incentives towards trust and support in dense networks (Crossley, 31). Importantly in our

case, however, networks in one specific social context may be dense, while the same actors may be involved in different, much less dense networks in other social contexts. One could enjoy a dense network of friends, but much looser networks at work or school. In doing our analysis of dense networks, we need to consider to what extent the actors were likely to have belonged to other important social groups, as well as to consider whether the survival pattern of documents was a major factor. It should be possible to speculate fruitfully whether what appears to be a very dense network of charter witnesses is likely to reflect a pattern of cohesion and embeddedness by weighing various factors.

The concept of homophily is typically used to characterise networks that have high densities. Homophily is sometimes described with the old adage 'birds of a feather flock together'. Sociologists have long recognised a tendency in humans to group together according to similar traits and tastes. It is harder to determine whether the groups form because of the actors' similarities, or instead whether actors who are already tied become more similar due to the effect of the ongoing social relationships. Sticking with the avian metaphor, this is considered the 'chicken and egg' problem of homophily. It is possible to study homophilous ties in social networks in a quantitative way, but this necessitates one's network having attributes which are measureable and relevant (Prell, 129-30). For medieval Scotland, it is hard to come up with such quantifiable attributes, but it is still worth asking whether homophily is relevant. Obviously, charter witnessing is a very particular kind of social relationship. In many cases, the witness himself will have little-to-no agency in the matter, particularly where charters were produced at large political assemblies. At the same time, there is a certain homophily involved in such cases, as witnesses will all be those deemed important or prestigious enough to witness a major royal charter, for example. In other cases, typically more local in nature, another kind of homophily might exist, one based on personal relationship, geography, and local community. As far as ego-networks go, it might be worth considering whether networks of greater density are characterised by a more homogenous group of documents and social contexts, and, by contrast, if the less dense networks involve more heterogeneous collections of documents. For example, an actor who has witnessed documents only having to do with episcopal properties in east Fife is more likely to have a network of homophilous cowitnesses, than would an actor who has witnessed royal charters, as well as those of an earl, a bishop, and an abbot.

We can test this by comparing the ego-networks of roughly equal size but with divergent ego-net densities. Walter Stewart (II), son of Alan (d. 1241) [1378] has a degree of 253, while Hugh of

Roxburgh, bishop-elect of Glasgow (d. 1199) [820] has a degree of 255. They also witnessed almost the exact same number of documents, 101 for Walter and 99 for Hugh. Walter had a much lower density, at 10.3, compared to Hugh, at 19.94. So what is different about the two networks? 81 of the 99 documents (81.8%) witnessed by Hugh were charters of King William (a further three were charters of the king's brother and son); this is not surprising given that Hugh was the king's chancellor. While Walter also witnessed a number of royal charters (71 out of 101), he also witnessed the charters of a number of different lay magnates and lords in the kingdom's southwest, especially the earls of Lennox. This explains a good deal of why a much larger number of co-witnesses from quite different social contexts appear in Walter's network. Another way of looking at it is to ask how many roles/ positions each person held, and how distinct they were from each other. Walter witnessed charters in his position as the king's steward, both in the king's household and in terms of being a major landholder in the Firth of Clyde region, but also as justiciar of Scotia in the 1230s. While Hugh of Roxburgh was briefly archdeacon of St Andrews, and achieved the rank of bishop-elect of Glasgow before his death, most of his career was spent as the king's clerk and chancellor. It is likely that even many of the non-royal charters that he witnessed were in his capacity as king's chancellor. He would have been surrounded by many of the same actors in these settings. In another example, Gregory, bishop of Dunkeld (d. 1169) [149] had a similarly sized ego-network to Henry, archdeacon of Dunkeld (fl.1183×1203-1220×25) [2762], 174 and 175 respectively. Archdeacon Henry's ego-net density (9.69) is dramatically lower, however, compared to that of Bishop Gregory (30.32). Sixty percent of Gregory's attestations were of royal documents, and a further 29 percent were documents of the bishops of St Andrews. While these appear to be two distinct categories, which we would normally associate with a more diverse body of witnesses and thus a lower density, at the time when Gregory was active, both royal and episcopal charters were often produced at large political assemblies attended by the same core group of actors. Indeed, players who were active in the middle of the twelfth century often have higher than usual densities. [124] Cospatric, earl of Dunbar (d.1166)'s density was 32.78, [172] Ranulf Soulis, king's butler (d.1165×72)'s was 28.72, and [88] William, abbot of Holyrood (I) (d.1172)'s was 40.32, to give but a few examples. So Archdeacon Henry's lower density is partly a result of the fact that he was active in the late twelfth and early thirteenth centuries, when the social contexts around document production were more manifold. Indeed, none of Henry's attestations were of royal charters. Despite the fact that 73 percent of his attestations were of ecclesiastical (H2) documents, most of which were based in his own home institution of Dunkeld, Henry witnessed in enough other social contexts to have a remarkably low-density network. This includes involvement in private transactions in central Scotland

and even Berwickshire, an area where the diocese of Dunkeld had far-flung possessions. The job of archdeacons, which involved representing the diocese's interests out in the world, on the ground, is a big part of why archdeacons often have ego-net densities on the lower end of the spectrum. [835] Laurence of Thornton, archdeacon of St Andrews (d. 1238×40), for example, had a density of 10.99. Members of the episcopal household whose job did not involve leaving the 'bubble' of the home institution, on the other hand, tend to have much higher densities. An example is [3057] Ralph, clerk of Bishop Roger of St Andrews, whose density of 34.09 belies the fact that he only witnessed documents of his employer or of St Andrews priory. Ralph's colleague [3072] Richard, chaplain of Bishop Roger, had an ego-net density of 33.3. Similarly, [167] William, chaplain (II) of King William (c.1196-1214) had a relatively high density of 25.57. Even though only 69% of the documents he witnessed were royal documents, most of the remainder of his attestations were private charters produced at the royal court or otherwise in a royal context where the king was present. While there is no shortcut to this kind of analysis, we can at least ask whether alters witnessed royal or other types of documents and display this in the sociogram (more on this below).

The conceptual world of low-density networks

Ron Burt's concepts of brokerage and structural holes are very important for our understanding of lowdensity ego-networks. Burt argues that some individuals act as 'brokers', holding a high level of influence or power due to their position in the network structure. In particular, they are well-placed to fill what Burt calls 'structural holes', or empty spaces between actors where the potential for meaningful connections exist (Prell, 122-24). This is based in part on Granovetter's notion of 'transitivity', whereby 'two nodes are more likely to have a tie when they each have a tie to a common third party' (Crossley, 15-16; 35). The broker would be the common third party, and might be in a position to benefit from bringing together the other two nodes in the triad. The sociological and psychological foundations for the role of the broker exist outwith Burt's SNA theories. Brokers are rich in what Robert Putnam calls 'bridging capital': by acting as bridges between different communities, brokers can facilitate the flow of ideas, resources, innovations, but also sometimes negative things like pathogens (Crossley, 31). In social network terms, brokers fill structural holes, acting as conduits between two otherwise distinct components, clusters or cliques. The particular structure of the network and position of the broker can also be important, and scholars have identified various types of brokers, including representative brokers, gatekeeper brokers, and liaison brokers, reflecting various arrangements of personal agency (Prell, 127).

Figure 8.6. Ego-network of [1336] Master Merlin (fl.1161x62-1189x1203)

This sociogram of the ego-network of Master Merlin, who flourished from the early 1160s until around the turn of the century, is illustrative of the bridging position of a broker in an ego-network. As we can see, Merlin acts as the main point of contact between two otherwise very dense components. Some of the actors in the segment on the right were [13] Earl Duncan II of Fife (d. 1204); [2] Matthew, bishop of Aberdeen (d. 1199); [202] Andrew, bishop of Caithness (d.1184); [139] William, bishop of Moray (d.1162); [141] William, abbot of Melrose (fl.1159-70); [149] Gregory, bishop of Dunkeld (d.1169); [226] Merleswain, son of Colban, lord of Kennoway (fl.1150s-90s); and [238] Malcolm, earl of Atholl (d.c.1197). Some of the actors in the segment on the left were [2115] Walter Lindsay (III), son of William (II) (d.c.1222); [3097] Richard Niger/Brun (12/13C); [5339] Reginald of Little Reston (brother of Aldan); [6180] Maurice, son of Master Merlin; [8099] Richard, forester (BWK); [8103] Constantine (at Ayton); and [9427] Philip, porter (BWK). It is clear that the actors on the right were powerful figures on the 'national stage', while those on the left were active in the local world of eastern Berwickshire. Master Merlin had ties with the episcopacy of Arnold, bishop of St Andrews, but was also likely a landholder in Berwickshire, where he witnessed a number of charters, and where his sons held land (PoMS, no. 1336). Indeed, there was a place called Merlington in the area which is now lost. So Master

Merlin was an individual with dense ties in the community of Berwickshire who also had connections to the more powerful in the kingdom, likely due to his higher education and ecclesiastical acumen.

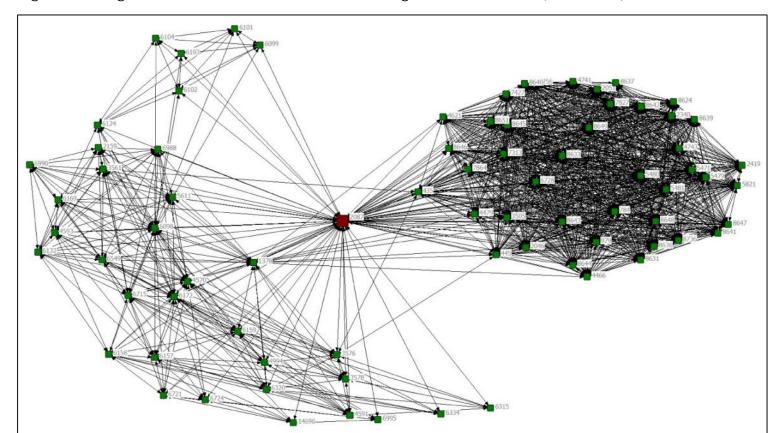


Figure 8.7. Ego-network of [2087] Mael Domnaig, earl of Lennox (d. ×1265)

Another example of an actor who is filling a structural hole in an ego-network is [2087] Mael Domnaig, earl of Lennox. As is obvious from his sociogram, the earl acts as the main connector between a very dense segment of actors on the right and a less dense segment on the left. The dense group on the right, however, is the result of a single document, the 1237 Treaty of York, witnessed by a large number of magnates and prelates, while the group on the left are witnesses to various charters dealing with local Lennox issues. While this serves as a salutary reminder that we must always follow up the documents underlying the SNA patterns, this sociogram does still represent the very real situation that earls could serve as bridges between the kingdom's 'national' politics and the locality of his own province.

Table 8.5. Fifteen actors with lowest ego-network densities

Poms ID	Name	Egonet Density	Degree/ size
3350	Adam of Makerstoun, master, provost (d.1280×86)	7.99	155
42	William del Bois, chancellor (d.1232)	8.04	476
13	Duncan (II) earl of Fife (d. 1204)	8.4	585
2190	Robert Mowat, knight, justiciar, sheriff of Forfar	9.43	153
2762	Henry, archdeacon of Dunkeld (fl.1183×1203-1220×25)	9.69	175
5364	John Cameron, sheriff of Perth	9.76	136
43	John, abbot of Lindores (fl.1219-44)	10.05	159
788	Andrew Murray, bishop of Moray (d.1242)	10.17	273
4427	William, son of Earl Patrick (I) (d.1253)	10.29	140
1378	Walter Stewart (II), son of Alan (d.1241)	10.3	253
2	Matthew, bishop of Aberdeen (d.1199)	10.4	473
858	Walter of St Albans, bishop of Glasgow (d.1232)	10.58	380
2067	Gilbert Hay (I), lord of Errol (d.1263) (son of David)	10.7	137
3432	Thomas Crook, knight	10.79	110
40	William Malveisin, bishop of St Andrews (d.1238)	10.82	379

A better way of considering possible brokers is to examine actors with low ego-net densities. While their brokerage opportunities may not be as visible in sociograms as the above examples due to the large size of many of their networks, the low density itself is a very important indicator. If denser networks are associated with social cohesion, support, and security, less dense networks are associated with competition and effectiveness. 'Relationships with insiders are more dense, supportive, and trusting, while relationships with outsiders are less dense and therefore open to manipulation afforded by "structural holes".' (Kadushin, 62). Individuals with the lower ego-net densities (Table 8.5) should be further investigated for potential broker status. This is because very few of the contacts with whom they have co-witnessed have also co-witnessed with each other. In other words, if charter witnessing were a proxy for knowing someone, only about 8-10% of the 'friends' of the people listed in Table 8.5 'know each other'. These actors are the common thread tying together various divergent groups of actors. That would present a great number of opportunities for these potential brokers to bridge those structural holes in meaningful ways.

The method of comparison might be useful for us here. Comparing a person with a lower-than-usual density with a person with higher-than-average density in the context of potential cohesion and safety

versus brokerage and effectiveness might be illuminating. While [202] Andrew, bishop of Caithness (d. 1184) had a large ego-network of 260 actors drawn from 88 documents, his density of 19.98 is 5.5 above the average for his degree range. 78 percent of the documents he witnessed were royal, and 16% were documents of the bishops of St Andrews. So despite his clearly very important position, he seems to circulate in a fairly homophilous environment. As we shall see, this could mean that a person like Andrew was able to draw safety, security, and strength from his highly embedded position in the tight-knit networks of the king's court and ecclesiastical capital, but he may not have been in a position to act as an opinion leader, or to introduce new ideas or energy into the dense network. His younger contemporary [2] Matthew, bishop of Aberdeen (d. 1199), who was also archdeacon of St Andrews from around 1150 to 1172, serves as a good counterpoint. Matthew witnessed 152 documents, and had a very large ego-network size of 473 and a very low density of 10.4. Some of Matthew's low density as compared to Andrew can be explained by his living at a slightly later date, when there were both more charters produced and more different types of social context producing them, tying into the fact that Matthew witnessed many more documents than did Andrew. Nevertheless, only 54% of Matthew's witnessing acts were royal charters, while another 30% were ecclesiastical grantors, most but not all of which related to St Andrews. But 14% of his co-witnessing was of private charters. A few of these would have been produced in a royal setting, but many were not. They included charters of many of the top landholders in Scotland proper at that time, including the earls of Atholl, Fife, Mar, and Buchan, the lords of Leuchars, and members of the Valognes, Maule, Avenel, Uviet, and de Fréville families. Remarkably, these activities were not a result of his position as archdeacon of St Andrews, but date to his time as bishop of Aberdeen, despite the fact that most of them relate to lands outwith the diocese of Aberdeen. What Matthew's low density is pointing to here is that he had an importance which is not fully captured by simply his titles and positions. Matthew was the leader of an important network which carried on the legacy of Bishop Robert of St Andrews, and it was likely this role combined with presumably effective personal abilities which made him a person for whom the whole is greater than the sum of its parts. Bishop Matthew should be seen as a very good contender for a broker in Scottish society. What this approach can give historians more broadly is the method of thinking about important actors according to the social groups they represented and various roles they filled, and how they may have acted as brokers or mediators between different interest bases in society. These interest bases could be particular provincial communities, religious houses or orders, international or more local kinbased networks, the royal house, burghs and trading networks, and so forth.

It is worthwhile pausing to consider the variations in network size between Bishop Matthew (473) and Bishop Andrew (260) and to question how much this affected the above analysis. This is particularly relevant in the context of how many documents were witnessed. Did the fact that Bishop Matthew witnessed more documents (152) than Andrew (88) determine his network size and thus the overall analysis? Clearly, the more documents one witnesses, the more opportunities there are to co-witness with new actors. First, Bishop Matthew's degree is 1.82 times that of Andrew, and Matthew witnessed 1.72 times as many documents, so there is a fairly close correlation there. Second, if we divide the degree (ego-net size) by the number of documents witnessed, we get an indicator of repetition of actors among the body of witnesses. This gives us a number of 3.11 for Bishop Matthew and 2.95 for Bishop Andrew. These indicators are broadly comparable, which suggests that the number of documents has not been the determinative factor in this case.

Table 8.6. Lowest degree/ documents witnessed ratios

Person name	ID	Degree/documents	Degree	Documents
William of Mordington	3673	1.943661972	138	71
Henry Balliol (d. 1246) [chamberlain]	1420	2.068965517	120	58
Richard de Moreville, constable (d. 1189 or 90)	112	2.094017094	245	117
Nicholas of Roxburgh, chancellor (d. 1171?)	133	2.123966942	257	121
Walter Barclay, chamberlain (d.c.1193)	6	2.172413793	189	87
Gilbert of Lumsdaine	3660	2.177419355	135	62
Philip de Valognes, chamberlain (d.1215)	15	2.186746988	363	166
William Comyn, earl of Buchan (d.1233)	16	2.193103448	318	145
Walter Comyn, earl of Menteith (d.1258)	1357	2.223404255	209	94
Robert, son of Gregory steward of Coldingham	7960	2.225806452	138	62
Hugh de Moreville (I) (d.1162) [constable]	79	2.305882353	196	85
William del Bois, chancellor (d.1232)	42	2.356435644	476	202
Adam of Prenderguest	6190	2.433333333	146	60
Walter Stewart (II), son of Alan (d.1241)	1378	2.504950495	253	101
Walter de Bidun, chancellor (d. 1178)	78	2.506024096	208	83
Walter son of Alan, steward (d. 1177)	3	2.532258065	314	124

First, there are a number of individuals on this list who are part of the Coldingham group, a corpus of critical mass which tends to feature the same actors with great regularity. These have been put in italics. When we remove these actors, there are clear patterns to the rest of the list. Nearly all of the players with very low degree/document ratios were royal household members, especially constables,

stewards, chancellors, and chamberlains. The only remaining two in the 'top 16' presented here were William Comyn and Walter Comyn, men who had similarly central positions at the royal court. This means that there were only about 2 unique individuals in their networks for each document they witnessed. Even though 7 individuals here witnessed more than 100 documents, their low ratios mean that they witnessed alongside the same people again and again, rather than encountering more new people. It makes a great deal of sense that people who witnessed mostly royal charters would have such low ratios. [13] Duncan (II), earl of Fife (d. 1204), despite witnessing 202 documents and having the largest ego-network size of 585, has a slightly higher ratio of 2.9. The people on the higher end of the spectrum tend to have witnessed much smaller numbers of documents, but the middle range is quite interesting. For example, [40] William Malveisin, bishop of St Andrews (d. 1238) witnessed 67 documents and had a ratio of 5.7, while [850] John Scot, bishop of Dunkeld (d. 1203) also witnessed 67 documents but had a lower ratio of 5.03. So even though they witnessed the same number of documents, Bishop William had more contacts (379) than Bishop John (337). Interestingly, Bishop William has a lower ego-net density (10.82) than Bishop John (13.34).

Not all structural holes are equally important, however. The two components must have something meaningful to offer: creating a bridge between two actors who are already very similar may ultimately be of little value (Crossley, 36). Burt developed a number of statistical measures to help fine-tune our analysis. Two of these are effective size and efficiency. Burt and Granovetter both accept that some ties are 'redundant' because they fail to bridge any meaningful gap. Nodes in a dense network which do not allow any new information to reach ego are redundant (Crossley, 36). Effective size is a modification of the measure of the ego-network size, the degree centrality or number of ego's contacts, which is calculated as ego's degree minus the average degree of the alters, not including ties to ego (Crossley, 83; Borgatti, 274-5). Another measure is efficiency, which is effective size divided by degree. The term 'efficient' here should be thought of in terms of broker expending time and resources on alters who themselves are unconnected. High efficiency means avoiding redundancy (Crossley, 91). Table 8.7, below, gives the effective size and efficiency calculations for the fifteen largest ego-networks, and table 8.8 gives the measures for the twenty lowest-density ego-nets. As we see, there is a general trend, but not an exact correlation, for the larger networks to be more efficient. While Duncan (II), earl of Fife has 585 contacts, the effective size of his ego-network is 536, giving him an efficiency of almost 92%. This is the inverse of his density, at 8.4%. Theoretically, about 536 of his 585 alters are unconnected, giving him ample opportunities for brokerage.

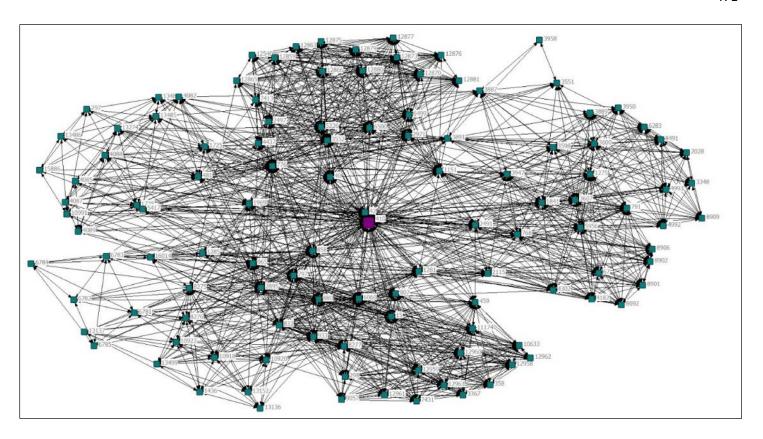
Table 8.7. Effective size and efficiency, 15 largest ego-nets

Person	ID	Density	Size	Effective size	Efficiency
Duncan (II) earl of Fife (d. 1204)	13	8.4	585	536	0.916
William del Bois, chancellor (d.1232)	42	8.04	476	437.8	0.92
Matthew, bishop of Aberdeen (d.1199)	2	10.4	473	423.9	0.896
David, earl of Huntingdon (d. 1219)	142	13.12	409	355	0.87
Walter of St Albans, bishop of Glasgow (d.1232)	858	10.58	380	339.9	0.894
William Malveisin, bishop of St Andrews (d.1238)	40	10.82	379	338.1	0.892
Malcolm (I), earl of Fife (d.1229)	782	11.54	377	333.6	0.885
Philip de Valognes, chamberlain (d.1215)	15	13.57	363	313.9	0.865
Jocelin, bishop of Glasgow (d.1199)	745	12.06	356	313.2	0.88
Gilbert, earl of Strathearn (d. 1223)	260	13.81	354	305.3	0.862
Richard de Prebenda, bishop of Dunkeld (d.1210)	798	15.39	347	293.8	0.847
Patrick (I), earl of Dunbar (d.1232)	444	12.56	343	300.1	0.875
John Scott, bishop of Dunkeld (d.1203)	850	13.34	337	292.2	0.867
Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	1285	11.14	327	290.6	0.889
William I, king of Scots (d.1214)	1	16.56	321	268	0.835

Table 8.8. Effective size and efficiency, 20 lowest density ego-nets

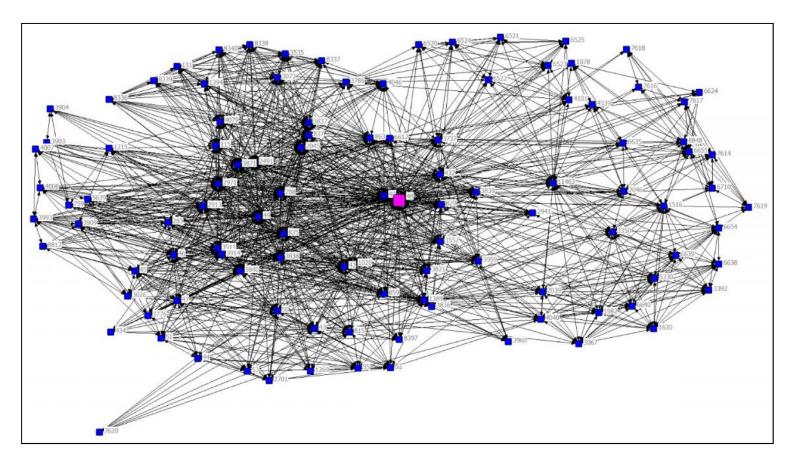
Person	ID	Density	Size	Effective size	Efficiency
Adam of Makerstoun, master, provost (d.1280×86)	3350	7.99	155	142.7	0.921
William del Bois, chancellor (d.1232)	42	8.04	476	437.8	0.92
Donnchad (II) earl of Fife (d. 1204)	13	8.4	585	536	0.916
Robert Mowat, knight, justiciar, sheriff of Forfar	2190	9.43	153	138.7	0.906
Henry, archdeacon of Dunkeld (fl.1183×1203-1220×25)	2762	9.69	175	158.1	0.904
John Cameron, sheriff of Perth	5364	9.76	136	122.8	0.903
John, abbot of Lindores (fl.1219-44)	43	10.05	159	143.1	0.9
Andrew Murray, bishop of Moray (d.1242)	788	10.17	273	245.3	0.899
William, son of Earl Patrick (I) (d.1253)	4427	10.29	140	125.7	0.898
Walter Stewart (II), son of Alan (d.1241)	1378	10.3	253	227	0.897
Matthew, bishop of Aberdeen (d.1199)	2	10.4	473	423.9	0.896
Walter of St Albans, bishop of Glasgow (d.1232)	858	10.58	380	339.9	0.894
Gilbert Hay (I), lord of Errol (d.1263) (son of David)	2067	10.7	137	122.4	0.894
Thomas Crook, knight	3432	10.79	110	98.2	0.893
William Malveisin, bishop of St Andrews (d.1238)	40	10.82	379	338.1	0.892
Bernard of Hadden, sheriff of Roxburgh	880	10.88	226	201.5	0.892
John Maxwell, chamberlain, sh. Roxburgh (d.1241)	1281	10.89	277	247	0.892
Laurence of Thornton, adcn. St Andrews (d.1238×40)	835	10.99	233	207.5	0.891
Alan of Harcarse, knight	5954	11.05	121	107.7	0.89
Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	1285	11.14	327	290.6	0.889

Figure 8.8. Ego-network of [410] Richard of Ancrum, dean, official, persona (fl.1202-26)



Before moving on, it might be worthwhile to glance at a couple of examples of actors whose structural position in the network gives them a distinct lack of influence. These are individuals who, through their position, have very low opportunity to fill structural holes, and who could be very easily replaced were they to disappear suddenly from the network. As is evident from the sociogram above, [42] William del Bois, chancellor (d. 1232) occupies a nearly identical position in the network structure to Richard of Ancrum himself. It is likely that William del Bois could subvert any brokerage opportunities of Richard of Ancrum if he wanted. A similar situation is shown in Figure 8.9, below, where [48] Simon de Noisy, clerk if Bishop William of St Andrews, is very closely situated to [49] William of Gullane, rector of Gullane. These are individuals who are on the opposite end of the spectrum to people with very efficient ego-networks. Instead, they are at high risk of redundancy.

Figure 8.9 Ego-network of [48] Simon de Noisy, clerk of Bishop William of St Andrews



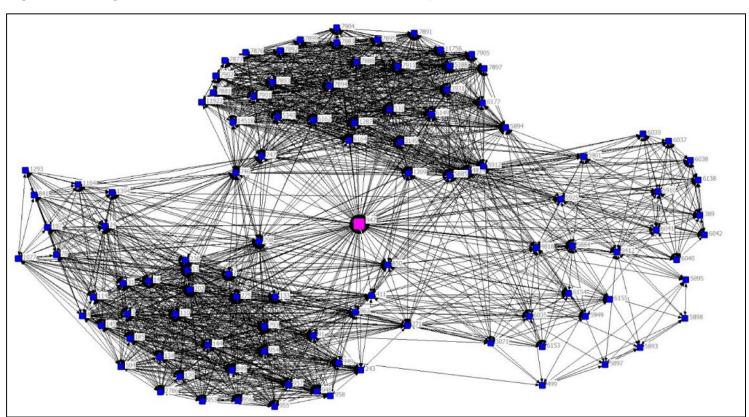
The strength of weak ties

Ron Burt's work on structural holes and brokerage was preceded by, and informed by, that of sociologist Mark Granovetter, on 'the strength of weak ties'. Granovetter's very influential work adds more nuance to our understanding of ego-networks by exploring the ways in which some contacts are different than others. Close friends, usually expressed in a dense network or network component, are described as strong ties, while mere acquaintances or people one only encounters infrequently are described as weak ties. While ego and his strong ties interact closely and are likely to possess a lot of the same information, have the same tastes, and so forth, ego's weak ties will themselves have strong ties with other actors who are not even part of ego's network at all. In this way, Granovetter's weak ties have a bridging function not unlike Burt's brokers. Weak ties move in different circles. New information is likely to enter a dense network through these weak ties (Kadushin, 30-1; Granovetter 1973 and 1983, Crossley, 35). In Granovetter's words, 'the fewer indirect contacts one has the more encapsulated he will be in terms of knowledge of the world beyond his own friendship circle' (Granovetter 1973, 1371.) Furthermore, weak ties are very important role in the integration of broader social systems; they are the glue that bonds together various more isolated and fragmented groups (Kadushin, 31). Without

them, according to Granovetter, 'subgroups separated by race, ethnicity, geography, or other characteristics will have difficulty reaching a modus vivendi' (Granovetter 1983, ??). Given the multi-ethnic and multilinguistic nature of society in central medieval Scotland, this concept could be invaluable to historians.

In our analysis of ego-networks, the work of Mark Granovetter has given us two key ideas which add nuance to our study. The first is that some parts of a network are more dense than other parts. There are areas of high and low density within the same sociogram. As one author puts it, 'the straightforward ego-net density measure is a relatively blunt structural instrument which fails to explore differentiation within the ego-net and which can disguise the fact that some parts of it are, in some cases, denser than others' (Crossley, 83). One method historians should consider is the identification of distinct components within the ego-network and asking whether they were likely to represent meaningful groups on the ground (Borgatti, 274). In small networks, such components may represent simply the witnesses of a single document. The following sociograms illustrate variations in density in ego-networks.

Figure 8.10 Ego-network of [947] Patrick, son of Cospatric, earl of Dunbar



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Figure 8.11 Ego-network of [37] Walter Murdoch

Figure 8.12 Ego-network of [562] William son of Thor, sheriff of Stirling

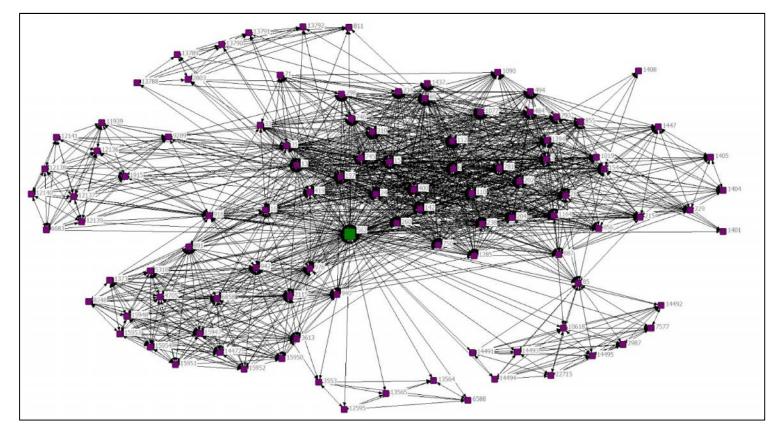


Figure 8.13 Ego-network of [831] Walter, abbot of Holyrood (d.1217 or 1218)

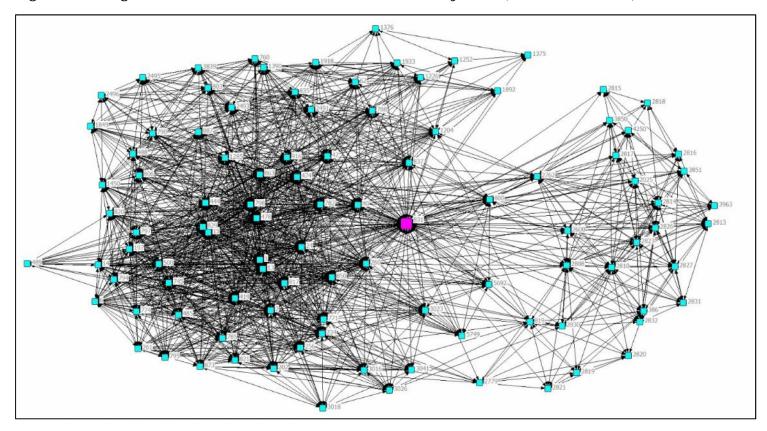
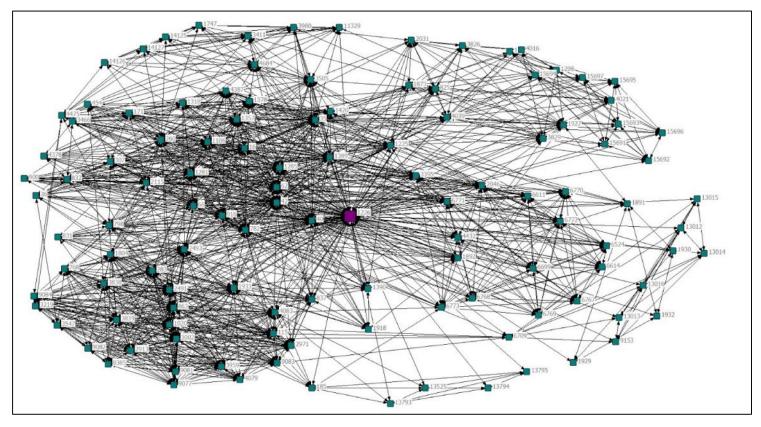


Figure 8.14 Ego-network of [1226] Geoffrey, son of Richard, of Inverkunglas, sheriff



The second idea that Granovetter gives us is that while all alters appear the same in our binary egonetworks, we need to add attributes and make them valued networks to give us a sense that some alters are strong ties and others are weak ties. The easiest way of doing this is to ask simply who witnesses many times with ego, and who witnesses only once or twice. The following series of sociograms show the ego-network of [13] Earl Duncan (II) of Fife (d. 1204), at various threshold levels of co-witnessing.

Figure 8.15. Ego-network of [13] Earl Duncan II of Fife (d. 1204)

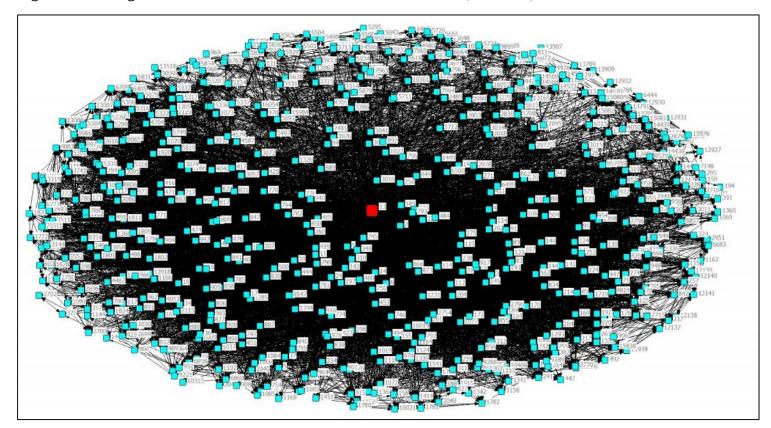


Figure 8.16. Ego-network of [13] Earl Duncan II of Fife (d. 1204), more than 5

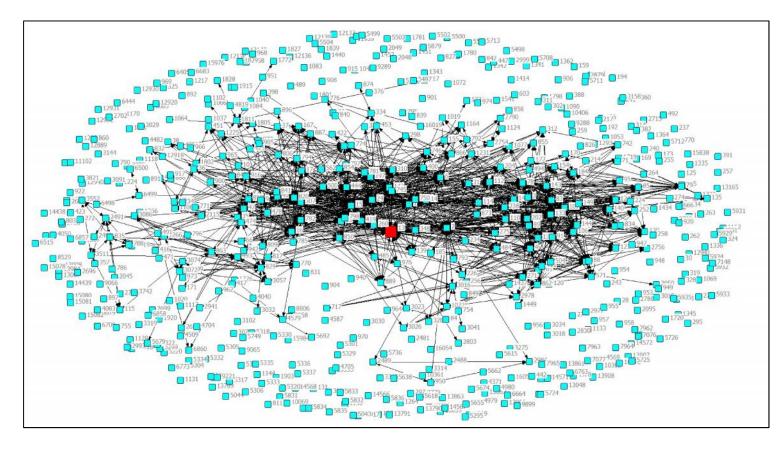


Figure 8.17. Ego-network of [13] Earl Duncan II of Fife (d. 1204), more than 10

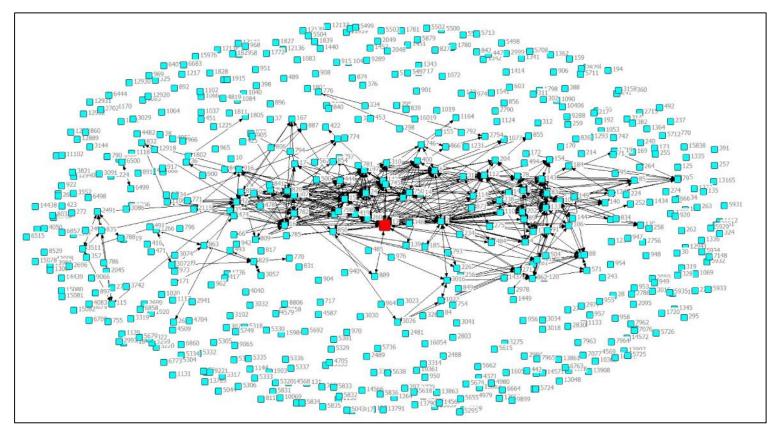


Figure 8.18. Ego-network of [13] Earl Duncan II of Fife (d. 1204), more than 20

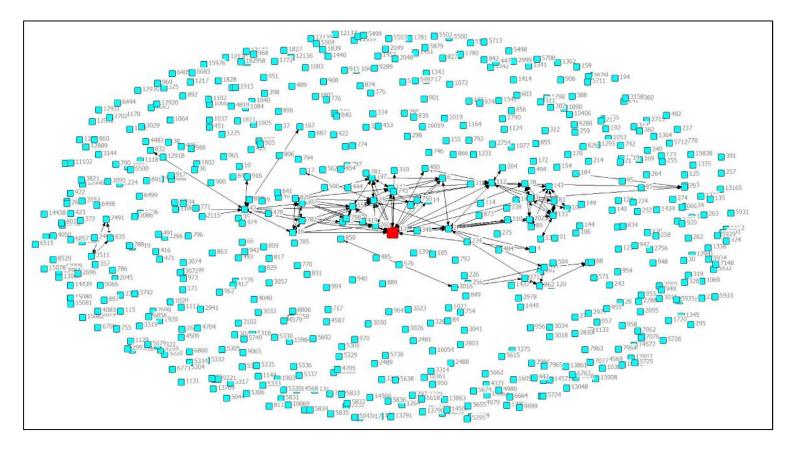
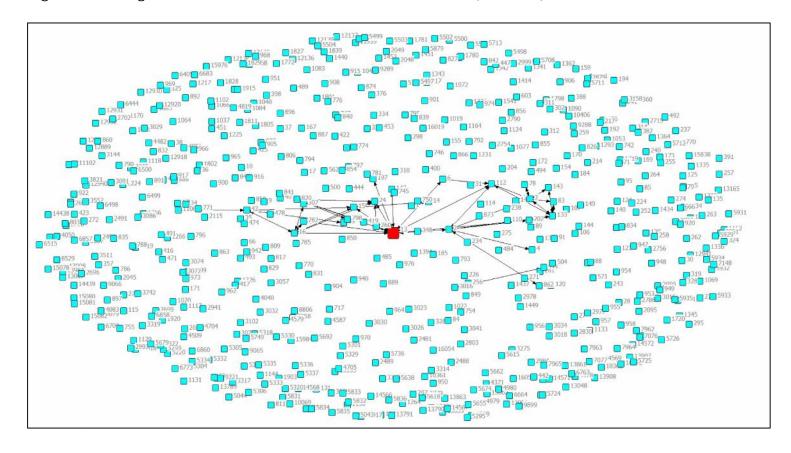


Figure 8.19. Ego-network of [13] Earl Duncan II of Fife (d. 1204), more than 30



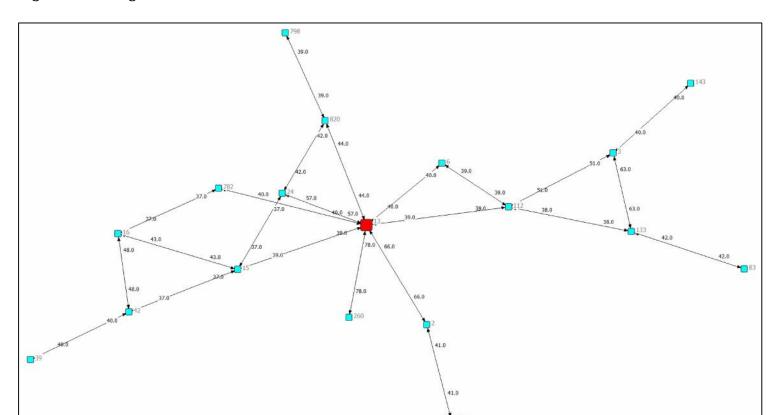


Figure 8.20. Ego-network of [13] Earl Duncan II of Fife, more than 35, with ties labelled

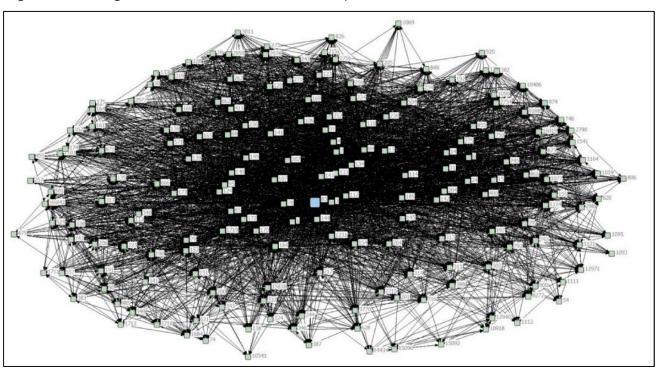
As the final sociogram in the series shows, Earl Duncan co-witnessed alongside eight people more than 35 times. The other people in the sociogram co-witnessed more than 35 times with each other but not as many times with Earl Duncan himself. The eight who witnessed the most with Earl Duncan were the following:

Table 8.9. Strongest ties of Earl Duncan (II) of Fife

Name	ID	# docs
Gilbert or Gilla Brigte, earl of Strathearn (d.1223)	260	78
Matthew, bishop of Aberdeen (d.1199)	2	66
William Hay (I), lord of Errol (d.c.1201)	24	57
Hugh of Roxburgh, bishop-elect of Glasgow (d.1199)	820	44
Walter Barclay, chamberlain (d.c.1193)	782	40
Malcolm (I), earl of Fife (d.1229)	6	40
Philip de Valognes, chamberlain (d.1215)	15	39
Richard de Moreville (d.1189 or 1190)	112	39

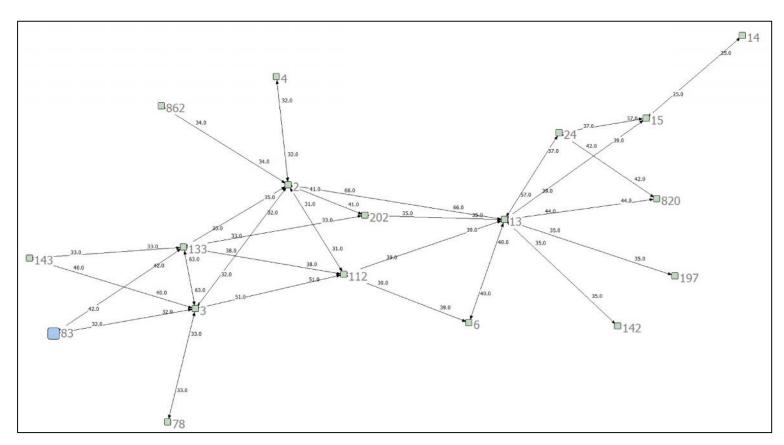
The above individuals are those with whom Earl Duncan had the strongest ties in the context of charter witnessing, and, given the high numbers of documents witnesses, most likely in life more generally. Of Earl Duncan's 585 contacts, 119 were people with whom he co-witnessed five or more times. These we might consider relatively strong ties. This means that 466 of Earl Duncan's contacts were people with whom he only co-witnessed four or fewer times. Indeed, he witnessed with 333 people only one time, and 82 people only two times. In theory, some of these weak tie co-witnesses could have acted as new sources of information and influence on Earl Duncan, while his closest contacts, like Earl Gilbert of Strathearn or William Hay of Errol, would have had less capacity for such influence. There is a paradox regarding the strength of weak ties, however, which is that research subsequent to Granovetter has shown that weak ties are much less important for actors who are already in a position of power, while weak ties provide opportunities for growth to those in a less advantageous position. Another somewhat paradoxical point is that Earl Duncan was himself in a position to be an 'opinion leader'. According to Kadushin, 'central individuals embedded in a system of strong ties not only have a high potential for transmitting ideas, but can also send messages to those who share those ideas or practices' (Kadushin, 145). So while the innovators who often introduce new ideas into a network are themselves less embedded, the influence makers and opinion leaders are, just like Earl Duncan, highly central (remembering Earl Duncan's paramount eigenvector centrality and very high betweenness in the whole graph) as well as deeply embedded.

Figure 8.21. Ego-network of [83] David Oliphant (12C)



[83] David Oliphant (12C) has a degree of 185, an ego-net density of 27.84, and witnessed 59 documents of the five specified types. As is visible in Figure 8.22, David's strongest ties, those he co-witnessed with the most, were the chancellor [133] Nicholas of Roxburgh (42 times) and the steward [3] Walter Stewart son of Alan (32 times). This is a reflection of the fact that David witnessed mostly royal charters, and these strong ties represent the security and trust involved in the dense world of the king's court. While David was part of this world, as the sociogram below shows, he was on its periphery. Moreover, 96 of David's contacts were people with whom he co-witnessed only once, and a further 24 were people with whom he witnessed only twice. While a detailed analysis of David's situation is beyond the remit of this book, it is among these weak ties that we would expect a man in David's situation to be able to make meaningful new contacts which might bring him new opportunities. In theory, weak tie alters should move in different circles. In David's case, the men at the royal court with whom he witnessed most often would be strong ties. Those who were less often at the king's court or were encountered in other social settings should be weak ties. These could be examined to identify the other circles in which they moved, by looking at who the strong ties were in their own ego-networks. This avenue of enquiry could be fruitful for the historian.

Figure 8.22. Ego-network of [83] David Oliphant (12C), more than 30, edges labelled



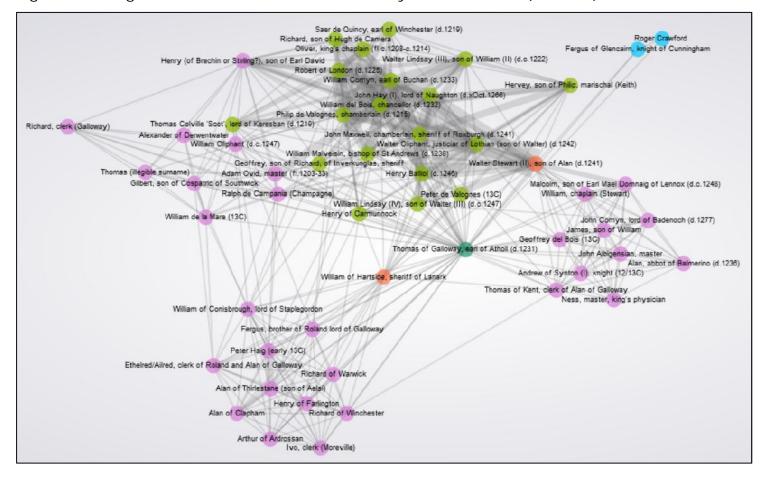
Finding the right mix

As should be clear from the above, there is no quick-and-easy shortcut for the historian wishing to find the most powerful or most influential actor in medieval society. Unfortunately, it is not as simple as asking who has the lowest density or highest efficiency. The social networks are never more than a reflection of the historical sources themselves, with all their inconsistencies in production, survival, and other factors. However, even with the best data possible, there is something of an art to analysing effectively ego-network data specifically and social network data more generally. As Brian Uzzi found in his study of New York City garment workers, embeddedness 'yields positive returns only up to a threshold point. Once the threshold is crossed, returns from embeddedness become negative... Optimal networks are not composed of either all embedded ties or all arms-length ties, but integrate the two'. (quoted in Kadushin, 68). Or, as Granovetter pointed out in 1973, a mixture of strong and weak ties can in some cases place ego in the best position (Crossley, 78).

One essential element to any analysis is that scholars always keep in mind the historical sources. The SNA data will largely reflect the variety and number of different social contexts reflected in the charters. While the situation is somewhat more nuanced than this, one relatively simple approach is to ask whether the actors appeared as witnesses in royal charters, or the charters of church prelates, lay magnates, or in the settlement of disputes. Because the H-number system used in the PoMS database employs these four basic categories, we can add as an attribute to the dataset whether witnesses appeared in only one of these contexts or in more than one. Witnesses who appeared in more than one context should be explored for potential as brokers or conduits.

Figures 8.23 through 8.26 show a Gephi sociogram of the ego-network of Thomas of Galloway, earl of Atholl (d. 1231), with his contacts colour-coded to reflect the document categories. It is also possible to highlight the nature of the links between actors with colour-coded edges or ties. Figure 8.24 shows how he co-witnessed in contexts which were royal, private, and settlement-related. Figures 8.25 and 8.26 show the connections of William of Hartside, sheriff of Lanark, and Walter Stewart (II), showing how they also witnessed across various contexts.

Figure 8.23. Ego-network of Thomas of Galloway, earl of Atholl (d. 1231)



Royal (H1) = green Private (H3) = pink

Settlements (H4) = blue Combination = peach

Figure 8.24. Ego-network of Thomas of Galloway, earl of Atholl (d. 1231)

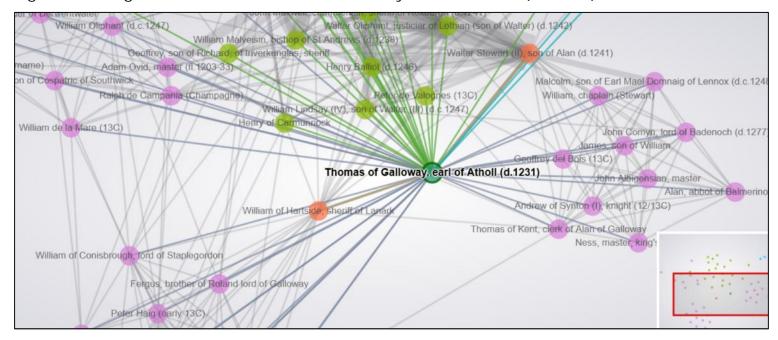


Figure 8.25. Ego-network of Thomas of Galloway, connections of William of Hartside

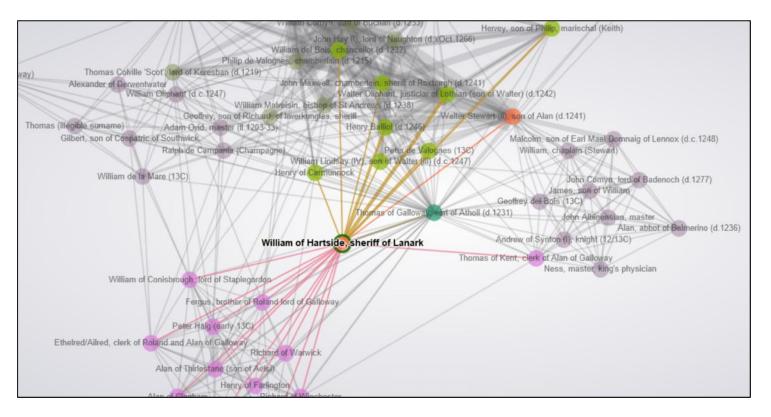


Figure 8.26. Ego-network of Thomas of Galloway, connections of Walter Stewart (II)

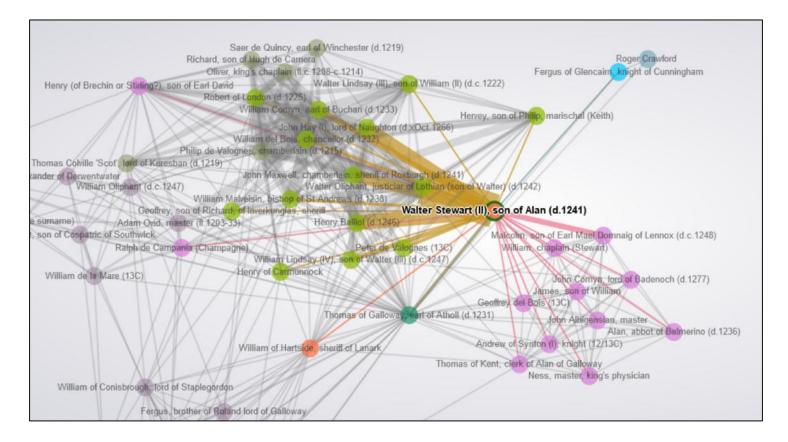


Table 8.10, which gives the 100 lowest density ego-networks of individuals with 100 contacts or more in the whole-network study of five specified document types in the PoMS database. The densities range from 7.99 up to 16.48, and there is a great variety of actors represented. In addition to the density and size, the table gives the number of documents (of the five specified types) witnessed by ego. This ranges from 202 on the upper end down to as low as 14. The final column tells us how many of these documents were non-royal. This is a quick shorthand to getting a feel for the social contexts in which the ego operated. Figures of 90% and above have been highlighted, but what we are really trying to separate out are the actors who operated in a very homogenous social context, versus those who operated in a range of social contexts. The same reservations could be made about those players who witnessed almost exclusively royal documents, such as Walter Stewart (9% non-royal), although more of these people have somewhat higher densities. Actors with very low densities with 90-100% non-royal witnessing probably offer historians less fruitful opportunities for further exploration. It is noteworthy that many of the actors identified elsewhere in this book as key figures tend to witness about 25-50% royal documents. The presence at the royal court suggests they were important on a 'national' level, but the critical mass of non-royal settings suggest they were not cocooned in the king's

presence. Some of these individuals included Duncan (II), earl of Fife (28% non-royal), Walter Stewart (II), son of Alan (d.1241) (30%), Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242) (26%), and Alexander Comyn, earl of Buchan (d.1289), justiciar (31%). It is also worth noting that most prominent bishops operate in the 60-80% range; see for example William Malveisin, bishop of St Andrews (d.1238) (61%), Jocelin, bishop of Glasgow (d.1199) (65%), Andrew Murray, bishop of Moray (d.1242) (67%), Gamelin, bishop of St Andrews (d.1271) (67%), and David of Bernham, bishop of St Andrews (d.1253) (78%). While this table offers a good place to start, it is really necessary to consider the four categories of documents used in the Gephi sociograms above, and then to dig deeper within those categories to identify and define specific social contexts.

Table 8.10. The 100 lowest densities

Rank	Name	PoMS ID	Density	Size	# docs	% non-
						royal
1	Adam of Makerstoun, master, provost (d.1280×86)	3350	7.99	155	28	100%
2	William del Bois, chancellor (d.1232)	42	8.04	476	202	40%
3	Duncan (II) earl of Fife (d. 1204)	13	8.4	585	202	28%
4	Robert Mowat, knight, justiciar, sheriff of Forfar	2190	9.43	153	40	80%
5	Henry, archdeacon of Dunkeld (fl.1183×1203-1220×25)	2762	9.69	175	26	100%
6	John Cameron, sheriff of Perth	5364	9.76	136	21	90%
7	John, abbot of Lindores (fl.1219-44)	43	10.05	159	23	92%
8	Andrew Murray, bishop of Moray (d.1242)	788	10.17	273	46	67%
9	William, son of Earl Patrick (I) (d.1253)	4427	10.29	140	28	93%
10	Walter Stewart (II), son of Alan (d.1241)	1378	10.3	253	101	30%
11	Matthew, bishop of Aberdeen (d.1199)	2	10.4	473	152	45%
12	Walter of St Albans, bishop of Glasgow (d.1232)	858	10.58	380	66	53%
13	Gilbert Hay (I), lord of Errol (d.1263) (son of David)	2067	10.7	137	29	24%
14	Thomas Crook, knight	3432	10.79	110	18	100%
15	William Malveisin, bishop of St Andrews (d.1238)	40	10.82	379	67	61%
16	Bernard of Hadden, sheriff of Roxburgh	880	10.88	226	24	54%
17	John Maxwell, chamberlain, sheriff of Roxburgh (d.1241)	1281	10.89	277	98	32%
18	Laurence of Thornton, archdeacon of St Andrews	835				
10	(d.1238×40)	833	10.99	233	61	100%
19	Alan of Harcarse, knight	5954	11.05	121	17	100%
20	Walter Oliphant, justiciar of Lothian (son of Walter) (d.1242)	1285	11.14	327	123	26%
21	Walter de Mortimer, dean of Glasgow (d.1270×71)	2044	11.38	109	33	100%
22	Malcolm (I), earl of Fife (d.1229)	782	11.54	377	103	35%
23	John of Huntingdon, master, official of Glasgow	776				
23	(fl.1179×1208)	770	11.62	251	35	97%
24	Alexander Comyn, earl of Buchan (d.1289), justiciar	1981	11.71	170	45	31%
25	John Hay (I), lord of Naughton (d.×Oct.1266)	1389	11.83	234	66	39%
26	Gamelin, bishop of St Andrews (d.1271)	456	11.9	143	18	67%

27	William of Ednam, master, archdeacon of Dunkeld (d.1251×57)	1969	11.97	181	29	100%
28	Henry of Stirling, son of Earl David	64	11.98	191	49	49%
29	Jocelin, bishop of Glasgow (d.1199)	745	12.06	356	77	65%
30	Walter Lindsay (III), son of William (II) (d.c.1222)	2115	12.11	293	62	63%
31	Luke, chaplain of Paisley	5071	12.23	148	19	100%
32	William of Greenlaw, master (d.1247)	768	12.29	164	26	100%
33	Philip Melville, justiciar of Scotia	1233	12.33	150	24	75%
34	David of Lochore, knight	2225	12.34	128	23	83%
35	Hugh, king's chaplain and clerk (episcopal witness)	2497	12.5	132	26	100%
36	Patrick (I), earl of Dunbar (d.1232)	444	12.56	343	73	52%
37	Adam, steward of Arbroath (son of Aldan)	4757	12.58	133	22	100%
38	Thomas Randolph (d.c.1296)	2139	12.66	132	23	56%
39	Simon, archdeacon of Glasgow (fl.1165×74-1195×96)	866	12.69	286	55	89%
40	Ralph de Lascelles, knight	5808	12.74	106	17	100%
41	William Francis (the Frenchman)	5226	12.75	140	14	100%
42	Patrick (II), earl of Dunbar (d.1248)	445	12.76	260	63	59%
43	Roger de Merlay (II) (d.c.1239), steward of Earl Patrick	5781	12.81	166	31	100%
44	William of Brechin, knight	2110	12.92	138	27	56%
45	Edward Murray, master, canon, bishop's clerk	3871	12.93	161	42	100%
46	David, earl of Huntingdon (d. 1219)	142	13.12	409	100	23%
47	Gregory, bishop of Brechin (fl.1189×98-1242×46)	59	13.13	135	34	100%
48	Alan of Thirlestane (son of Aelsi)	6499	13.19	172	21	100%
49	John Scott, bishop of Dunkeld (d.1203)	850	13.34	337	67	84%
50	Philip de Valognes, chamberlain (d.1215)	15	13.57	363	166	13%
51	Simon Fraser (d.1291×92)	1810	13.71	102	32	50%
52	Gilbert, earl of Strathearn (d. 1223)	260	13.81	354	97	30%
53	Alan Durward (d.1275)	1971	13.95	145	43	30%
	Bernard Fraser (in ELO and BWK) (brother of Ness and John					
54	of London)	11520	13.95	108	34	97%
55	John de Vaux, knight (fl.1213-55)	2081	14	147	40	43%
56	Peter Ramsay, bishop of Aberdeen (d.1256)	2047	14.15	110	18	89%
57	Robert Menzies (d.1267)	2065	14.19	151	44	32%
58	Patrick (III), earl of Dunbar (d.1289)	446	14.25	102	21	43%
59	Thomas de Normanville, knight	2328	14.55	113	19	74%
60	Andrew, archdeacon of Lothian (fl.1147×59-1178×84)	411	14.58	257	62	97%
61	David Graham, lord of Lovat (d.c.1272)	2005	14.71	133	23	87%
62	William Comyn, earl of Buchan (d.1233)	16	14.8	318	145	12%
63	Gilbert, archdeacon of Dunblane (fl.1203×10-1235×39)	466	14.83	115	28	100%
64	John of Stirling, knight, sheriff of Stirling	1228	14.95	118	25	64%
65	Walter son of Alan, steward (d. 1177)	3	14.96	314	124	9%
66	Elias of Partick, clerk, canon (son of Fulbert)	926	14.98	186	29	100%
67	Herbert, dean of Glasgow (fl.1179×89-1204×07)	481	15	207	32	100%
68	David Hay, lord of Errol (d.1237×41)	66	15.02	256	44	57%
69	William, earl of Mar (d.a.1281)	2041	15.06	130	38	29%
70	Peter Haig (early 13C)	6500	15.13	128	15	100%
71	Peter, chaplain and clerk of Bishop Malveisin	2971	15.16	178	40	100%
72	Ralph de Campania (Champagne)	3793	15.22	130	20	45%
73	Malcolm Lockhart (12C/13C)	4625	15.25	141	16	94%

74	Walter Stewart, earl of Menteith (d.c.1293)	2151	15.25	113	16	69%
75	Duncan, son of Earl Duncan (II) of Fife	1326	15.26	182	16	94%
76	Stephen of Lilliesleaf, master, clerk, persona	2491	15.3	179	40	100%
77	Geoffrey de Liberatione, bishop of Dunkeld (d.1249)	2039	15.3	156	34	38%
78	Richard Nanus (le Nain)	6060	15.32	165	26	100%
79	Richard de Prebenda, bishop of Dunkeld (d.1210)	798	15.39	347	82	29%
80	Freskin Douglas, dean of Moray (d.1226)	3761	15.45	106	21	100%
81	Alan, son of Cospatric of Swinton	1287	15.5	236	76	100%
82	Alexander, chaplain of bishops of St Andrews (12C)	3016	15.52	211	61	98%
83	Ranulf de Wat, archdeacon of St Andrews (d.1209)	829	15.55	277	45	100%
84	Walter Comyn, earl of Menteith (d.1258)	1357	15.55	209	94	13%
85	Aulay, brother of Earl Mael Domnaig of Lennox	4570	15.58	113	24	100%
86	Hugh de Sigillo, bishop of Dunkeld (d.1229 or 1230)	39	15.68	303	71	42%
87	William Lindsay (IV), son of Walter (III) (d.c.1247)	4425	15.68	167	51	51%
88	Ralph, king's chaplain (TRA2)	3576	15.74	124	31	55%
89	Reginald Crawford, sheriff of Ayr (early 13C)	1254	15.8	119	20	75%
90	Andrew, dean of Lothian/Tyninghame (fl.1194-1214)	414	15.89	182	19	100%
91	Hugh Barclay, justiciar of Lothian	2104	15.9	105	21	52%
92	Robert, archdeacon of Glasgow (d.1222)	797	15.97	233	52	44%
93	Robert Crook, knight (12C)	16019	16.03	187	20	95%
94	Malcolm Beg, son of Gillespie Galbraith	6172	16.21	121	21	100%
95	Aymer Maxwell (d.1266)	2091	16.28	117	23	39%
96	William of Hownam, son of John, son of Orm (d.1227)	933	16.4	137	20	95%
97	Henry, son of Geoffrey de Liberatione of Perth	5330	16.42	151	17	100%
98	John of Leicester, bishop of Dunkeld (d.1214)	493	16.43	241	30	100%
99	Aiulf, dean of Lothian (fl.1150/51-1186)	165	16.44	238	49	96%
100	David of Bernham, bishop of St Andrews (d.1253)	432	16.48	125	27	78%

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9 Using Agent-Based Modelling and Social Network Analysis to Gain Further Insights into the People of Medieval Scotland (by Cornell Jackson)

In order to get some more possible insight into the process of selecting charter witnesses in medieval Scotland, it was decided to build a model to seek to replicate the social networks based on charter witnessing during the 11th through 13th centuries in Scotland. In general, the plan was to create rules for the model to generate social networks that would be compared to an empirical network generated from the charter data in the database. The closer the match, the more probable that these rules were the ones actually used in medieval Scotland to select charter witnesses.

Agent-based modelling is a relatively new paradigm of system modelling (Monostori et al, 2006). A model is a representation of some real system (Starfield et al, 1990). According to Bonabeau (2002), an agent is a computational system that is situated in a dynamic environment and is capable of exhibiting autonomous and intelligent behaviour. Agents could interact, communicate and exchange information with each other. Sammarco et al (2014) describe the common properties of computational agents to include the following:

- Autonomy Agents act based on both their internal state and the behaviour of others in the environment.
- Intelligence Agents have some kind of intelligence from applying fixed rules.
- Interaction Agents are able to interact with their environment and other agents.
- Adaptation Agents adapt their behaviour to the changes of the environment based on their programmed intelligence.

Agents may represent people or technology, such as a workstation or machine, in the system being modelled. One key feature of agent based modelling is that in a multi-agent system, complex actions emerge from interactions among agents. So, a multi-agent system exhibits emergent behaviour that cannot be derived from individual knowledge but from the interaction and information exchange between many agents (Monostori et al, 2006).

Generally one starts simple when building an agent based model (Railsback and Grimm, 2012). It may be thought simple models may not provide much insight into real systems. However, the seminal studies by Thomas Schelling (1969, 1971), who won the Nobel Prize in Economics in 2005, was able to use agent based modelling to explain how racial segregation in the United States results from

moderate preferences for same race neighbours even if there is no preference for totally segregated neighbourhoods.

Agent-Based Modelling and Social Network Analysis

In recent years, there has been more and more research combining social network analysis and agent based modelling. This is especially useful when the focus is on more fine-grained chains of events want to be modelled and macroscopic consequences of network dynamics are the focus of the investigation (Manzo and Tubaro, 2016).

Manzo and Tubaro (2016) mention several fields that are using both social network analysis and agent-based modelling. These include strategic networks (Buskins et al, 2014) and specifying network topologies (Axtell, 2001). Also, epidemiology has used both social network analysis and agent based modelling to understand the contagion effect and the roles of reciprocity and feedback loops in the spread of disease (EI-Sayed et al, 2012).

Manzo and Tubaro (2016) note that there is tension between social network analysis and agent-based modelling on the emphasis placed on statistical estimation and causal inference citing Snijders and Steglich (2015). However, they feel there is much room for synergy between the two approaches that could significantly add to knowledge.

Charter Witness Selection Model Description

The purpose of this agent-based model is to simulate the process of selecting witnesses for medieval Scottish charters. The purpose of the witnesses was to testify if necessary in order to verify the charter.

In following the advice above to start simple when constructing a model, initially the building of the model focused on just one aspect. One key assumption is that the higher the status of the witness, the more effective the verification. Therefore, the key aspect that was concentrated on was status. The model assumes that the grantor of the charter would like to get the highest status witnesses that are available. The model steps through a series of witness types starting with the highest status ones. The model also assumes that the grantor would like to select the highest status person within a witness type. So, for example, the grantor would prefer the highest status bishop to be a witness if available.

Availability within the model is controlled by thresholds and a random number generator. Each witness type has a threshold of availability. The higher the status, the lower the threshold. The model generates a random number and if it is lower than the threshold this person is added to the witness list.

Below is a table that contains the witness types in status order along with the threshold value and the number of potential witnesses of each type. The potential number of witnesses are roughly based on a subset of the empirical co-witnessing network where all witnesses had at least 21 connections.

Table 9.1 Witness types and thresholds

Order#	Witness Type	Threshold%	#Potential Witnesses
1	King	5	1
2	Prince	5	1
3	Bishop	20	16
4	Chancellor	35	3
5	Archdeacon	40	16
6	Justiciar	40	6
7	Chamberlain	55	8
8	All others	70	100

Finally, the number of witnesses needed for each of the one thousand charters used was randomised.

Methodology

In general, the methodology used in this research generates networks from the model and compares these to an empirical network. A statistical technique is used to compare the networks and if the comparison is close enough to be considered statistically significant then there is a high degree of probability that the rules in the model are the same used by grantors in medieval Scotland.

The empirical network being used as a baseline is the co-witnessing network of all those witnesses that have at least 21 connections to other witnesses. It was felt that this network was easier for initial model testing rather than the full network of 9078 witnesses. Our subset of the full empirical network has 95 witnesses.

The statistical technique used to compare the two networks in social network analysis is the quadratic assignment procedure (QAP) correlation. We are using UCINET (Borgatti et al, 2002) to do this. This technique uses a traditional Pearson's correlation to see how the numbers in the two data matrices that represent the two networks move together or apart. There then is a test to see how much of the matching is due to randomisation. This technique requires that the two networks have the same number of witnesses. Therefore, only model generated networks containing 95 witnesses were compared to the empirical network.

Findings

The percentage of random correlations in the QAP correlation is called a p value. The smaller the p value, the better the match between the two networks. In statistics, traditionally only results that are considered statistically significant are deemed conclusive. The norm for statistical significance in the social sciences is a p value of 0.05 or lower. This is equivalent in this case of only 5% of the match between two networks is due to randomisation. However, other fields of research set the threshold for statistical significance higher with a p value of 0.1 or 10%. Based on advice from experts on statistical techniques, we set our threshold for statistical significance at 10%.

The model generated 23 networks that had 95 witnesses. These were compared to the empirical cowitnessing network of witnesses that had at least 21 connections with other witnesses. The p values ranged from 0.175 to 0.088. While the lower numbers meet the threshold for statistical significance, the average p value for all 23 networks was 0.12.

Conclusion and Next Steps

While this result is not statistically significant, it was surprising to see how close to statistical significance the model got by using only rules on status. This may not be a big surprise that status plays an important role in charter witness selection but it does show the potential for agent based modelling in understanding the past.

The next refinement to the model was to consider time. The charters were produced over a two hundred year period and witnesses lived and died within that time frame. The model will now give each charter an issue date and only witnesses alive at the time would be considered. All the previous rules on status still applied.

Unfortunately, the average p value increased to 0.33. This indicates the first draft of the new model needed more work but the project ended before this could be tackled. The hope is to be able to do this in the future.

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